A Hypothesis Concerning the Dynamic Balance of Fresh Water and Salt Water in a Coastal Aquifer

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Abstract—The dispersion of salts produced by reciprocative motion of the salt-water front in a coastal aquifer induces a flow of salt water from the floor of the sea into the zone of diffusion and back to the sea. The head losses that accompany the landward flow tend to lessen the extent to which the salt water occupies the aquifer.

Introduction—Published explanations of the steady-state balance between salt water and fresh water in a coastal aquifer, beginning with Badon Ghyben [1889] and Herzberg [1901], commonly assume that the salt water is static. Under this assumption the balance would be as shown in Figure 1. The depth below sea level to a point on the interface would be [Hubbert, 1940, p. 872]

\[ z = \frac{\rho_f}{\rho_s - \rho_f} h \]

where \( \rho_f \) is the density of the fresh water, \( \rho_s \) is the density of the sea water, and \( h \) is the head of fresh water above sea level at the point on the interface. An equation for determining the shape and position of the interface with a known rate of discharge of fresh water under one set of boundary conditions has recently been devised by Glover [1959] through an adaptation of a solution by Koszeny [1953] for an analogous problem of free-surface gravity flow. Equations for the interface under several sets of boundary conditions have been derived by H. R. Henry (in preparation).

It is the thesis of the present paper that where a zone of diffusion exists, the salt water is not static but flows perpetually in a cycle from the floor of the sea into the zone of diffusion and back to the sea, and that this flow tends to lessen the extent to which the salt water occupies the aquifer.

That this cycle must exist in some degree becomes evident if one considers that there is a continuous discharge of salty water from the zone of diffusion into the sea (Fig. 2). For if

![Fig. 1—Balance between fresh water and salt water in a coastal aquifer, with the salt water static](image-url)
there is such a discharge, the sea itself must be the source of the salts that are carried back to the sea, and hence the salts must by some means be transported from the floor of the sea through the aquifer into the zone of diffusion. We may rule out any form of dispersion or diffusion as the principal mode of transportation of the salts from the sea to the zone of diffusion on the grounds that these processes occur only when there is a concentration gradient, and the concentration gradient that exists across the zone of diffusion will not in general extend in sufficient magnitude all the way back to the sea floor. Therefore, we may conclude that the salts are transported largely by a hydraulic flow of the salt water, as indicated in Figure 2, with a consequent loss of head in the salt-water environment.

One example of a discharge of salty water from the zone of diffusion is the flow of springs
around Pearl Harbor, Hawaii. Ordinarily, most of the seaward discharge of water from a coastal aquifer occurs at the floor of the sea, but around Pearl Harbor a sizable quantity of water issues from terrestrial springs as the result of the presence of the 'caprock,' a relatively impervious blanket of weathered volcanic debris that underlies and rims the harbor and forces some of the ground water from the basaltic aquifer to discharge in springs along its inland perimeter. Typical of these is Waimano Springs (Fig. 3), consisting of several orifices that yield water of different salinities. The total flow of the several orifices of Waimano Springs ranges from 15 to 20 million gallons per day. The water from these orifices increases in salinity toward the sea, and chloride content ranges from less than 200 ppm at the orifices farthest inland to as much as 3200 ppm at the one nearest the shore. The chloride content of sea water at most places is about 19,000 ppm.

The great thickness of the zone of diffusion beneath the Pearl Harbor area is indicated by Figure 4, which shows the chloride content of water at various depths as determined from the drilling of a deep test well. The zone begins at about 200 ft and ends at perhaps 1200 ft, thus having a thickness of 1000 ft or more.

Another place at which there is evidence of discharge from the zone of diffusion is the Cutler area, near Miami, Florida. The zone of diffusion in the Biscayne aquifer of this area, which consists predominantly of cavernous limestone, is represented by the isochlors in Figure 5. The control points for the isochlors are shown by the black dots, which represent the bottoms of fully cased wells. These wells were drilled for the purpose of extracting water samples and measuring pressure heads at isolated depths, as a part of an investigation by the U. S. Geological Survey of the various factors relating to the hypothesis described in this paper. The thickness of the zone of diffusion is limited by the thickness of the aquifer, but its horizontal breadth at the base of the aquifer, as reckoned from the isochlors, is probably about 2000 ft. The zone extends several hundred feet beyond the shore line. The distribution of salts is doubtless influenced by the presence of beds of low permeability in the aquifer.

As most of the seaward discharge from the Biscayne aquifer occurs at the floor of the sea, there is no opportunity to obtain representative samples of the water being discharged to determine its salinity. We may infer, nevertheless, that salty water is being discharged from the fact that the shallow wells that end in the zone of diffusion beneath the floor of the sea tap salty water under sufficient head to rise above sea level.

Mechanics of the cyclic flow—But what causes the sea water to move in its cycle through the aquifer? Since the water has the same fluid potential when it re-enters the sea as when it leaves, one might reason that there can be no hydraulic gradient and hence no flow. To understand how a flow may exist under these circumstances, let us consider the processes that operate in the zone of diffusion. Sea water and fresh water become intimately mixed in the zone of diffusion by the mechanism that creates this zone. The effect of this is the same as if some of the salt ions were extracted from the sea water and injected into the flowing fresh water. The diluted sea water, having become less dense than native sea water, rises along a seaward path. The resulting circulation is analogous to the circulation in thermal convection, differing from it only in that changes in density are produced by changes in concentration rather than by changes in temperature. Meanwhile, the salts that are introduced into the fresh-water environment are carried back to the sea by the flow of the fresh-water system. (It is convenient to consider the salt-water environment as includ-
ing that part of the zone of diffusion from which there is an efflux of salts by dispersion, and the fresh-water environment as including the part into which there is an influx of salts by dispersion. The boundary between the two will then be the point of inflection of the concentration curve.)

Apparently, then, the circulation is induced by the transfer of salts out of the salt-water environment. The forces that effect this transfer must be powerful enough to recreate the zone of diffusion continuously as it is dissipated by the flow of mixed water to the sea. As molecular diffusion is much too feeble for this, we must look for something more potent.

Dispersion—A mechanism that appears to be powerful enough to cause sufficient mixing is the reciprocative motion of the salt-water front that occurs as the result of ocean tides and of the rise and fall of the water table due to variations in recharge and other forces, including pumping. It has been theorized by Palmer [1927, pp. 51–52] and Wentworth [1948] that it is this to-and-fro motion that creates the zone of diffusion. The process by which two miscible liquids interfuse about their boundary when the boundary is caused to move by hydraulic flow is known as dispersion. In laminar flow through permeable porous media, dispersion is produced by the combined effects of convection (transfer of a fluid into the region of another due to variations in velocities within the inter-
unity. They found the values of $M$ to be 0.063 for Ottawa sand and 0.13 (computed from their Table V; the value of 0.013 given in their Fig. 18 appears to be incorrect) for Monterey sand. Recent experiments by Orlob and Radhakrishna [1958] indicate that the medium dispersion constant $M$ increases with the uniformity coefficient of the medium and becomes as high as 2.79 for a sand having a uniformity coefficient of 3.88.

If the coefficient of longitudinal dispersion is practically proportional to the first power of the mean interstitial velocity, it may, in the case of dispersion due to ocean tides, be expressed

$$D = 4MA/t_0$$  \hspace{1cm} (1)

where $A$ is the amplitude and $t_0$ is the period of the displacement of water in the aquifer caused by ocean tides.

The amplitude of the tide-produced displacement of water may be related approximately to the amplitude of the tide and the distance from the shore line as follows. The tide-produced change in the artesian head with reference to its mean in a semi-infinite artesian aquifer is found to be [Jacob, 1949, p. 365; Ferris, 1951, p. 149]

$$h = h_o \exp \left( -x \sqrt{\pi S/t_0 T} \right)$$

$$\cdot \sin \left( 2\pi t/t_0 - x \sqrt{\pi S/t_0 T} \right)$$

where

- $h_o$ = amplitude of the tide
- $x$ = distance from the shoreline
- $t$ = time referred to the beginning of a tidal cycle
- $t_0$ = period of the tide cycle
- $S$ = coefficient of storage
- $T$ = coefficient of transmissibility.

The gradient producing the displacement will therefore be

$$\frac{\partial h}{\partial x} = -h_o \sqrt{\pi S/t_0 T} \exp \left( -x \sqrt{\pi S/t_0 T} \right)$$

$$\cdot \left[ \sin \left( 2\pi t/t_0 - x \sqrt{\pi S/t_0 T} \right) \right.$$

$$+ \cos \left( 2\pi t/t_0 - x \sqrt{\pi S/t_0 T} \right) \right].$$

The displacement will be zero at a time $t_1$ when $\partial h/\partial x$ is a maximum, or when the quantity in parentheses is $\pi/4 + n\pi$. It will be extreme at a time $t_2$ when $\partial h/\partial x = 0$, or when the quantity in parentheses is $3\pi/4 + n\pi$. Thus, with the substitutions

$$u = 2\pi t/t_0 - x \sqrt{\pi S/t_0 T}$$

$$du = 2\pi dt/t_0,$$

the amplitude of the displacement will be

$$A = K/\theta \int_{t_1}^{t_2} (\partial h/\partial z) \, dt$$

$$= \left( K h_o/\theta \right) \sqrt{t_0 S/4\pi T} \exp \left( -x \sqrt{\pi S/t_0 T} \right)$$

$$\cdot \int_{\pi/4}^{3\pi/4} (\sin u + \cos u) \, du$$

$$= \left( K h_o/\theta \right) \sqrt{t_0 S/2\pi T} \exp \left( -x \sqrt{\pi S/t_0 T} \right),$$  \hspace{1cm} (2)

where $K$ is the permeability and $\theta$ is the effective porosity of the aquifer.

With (1) and (2) one may estimate the coefficients of tide-produced dispersion at given distances from the shore line in a typical coastal aquifer, if it is assumed that the medium dispersion constants obtained from experiments with unidirectional flow are applicable, and that the displacements of the interface will be the same as the displacements that would occur if the water in the aquifer were all of one density. The latter assumption appears to be reasonable when we consider that in most aquifers the interface will move very slowly in response to changes in head and that its maximum displacement will be only a fraction of that which would be required for it to adjust completely to the extremes of the tide. Equation (2) was derived for artesian aquifers but probably will give a fair approximation of the amplitude of the displacement in a nonartesian aquifer if an appropriate value for the coefficient of storage is used.

The amplitude of the oscillation of water at various distances from the shore line in a hypothetical nonartesian aquifer of sand is illustrated in Figure 6. If the medium dispersion constant of the sand were 1.0, the coefficient of dispersion would be 100 cm$^2$/day at about 300 ft from the shore line, and 10 cm$^2$/day at about 900 ft from the shore line. Beyond 1500 ft it would not be significantly larger than 1 cm$^2$/day, which is approximately the coefficient of molecular diffusion of sodium chloride [Hodgeman, 1945, p. 1695].
It appears likely, however, that in aquifers that consist of alternating beds of high and low permeabilities, as practically all aquifers do, there is a mixing mechanism that will produce rates of dispersion considerably larger than those indicated by laboratory experiments on unidirectional flow through homogeneous sand. Suppose, for example, that we have an aquifer made up of alternating beds having permeabilities in the ratio of ten to one, and that initially there is a sharp interface lying diagonally across the beds. If, with a rise of the tide, the interface were to move landward a distance of \( x \) units in a bed of high permeability, it would move only about 0.1\( x \) units in an adjacent bed of low permeability. Consider, now, what would happen if the fresh water were then to begin flowing in a direction diagonally across the beds in an upward path toward the sea. Apparently, the water in the various beds would become fairly well mixed, so that there would result a zone nearly \( x \) units wide in which the concentration of salts would be some fraction of that of sea water, this fraction being dependent on the relative thicknesses and porosities of the beds. With each additional tidal cycle the zone would widen further until ultimately the distribution of salts would come to a steady state wherein the average rate at which the salts were carried into an element of the aquifer by dispersion would be in balance with the rate at which they were carried out of it by hydraulic flow. The rate of dispersion produced by this process would doubtless be greater in an aquifer of cavernous limestone or basalt than in one of laminated sand.

The reader will understand that the hydraulics is somewhat more complicated than postulated here. The example given is merely a device to illustrate the idea of dispersion by differential displacement and cross-bed mixing. The displacements and mixings will not, of course,
occur as separate, alternate events but will operate simultaneously much of the time, each playing a more prominent part at one phase in the tidal cycle than at another.

Summary—It appears to be reasonably certain that wherever a zone of diffusion exists in a coastal aquifer a flow of sea water from the floor of the sea into the zone of diffusion will occur. The flow may be interrupted or reversed during low stages of the tide or high stages of the water table, but on the average it will persist in a landward direction. Apparently, the only question in a given case is one of magnitude. The magnitude of the flow evidently will be governed chiefly by that of the dispersing mechanism that induces it, and it may in some places be large enough to produce head losses in the salt-water environment that would lessen appreciably the extent to which the salt water occupies the aquifer.

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