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COAP 2017 Best Paper Prize

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Each year, the editorial board of Computational Optimization and Applications (COAP) selects a paper from the preceding year's publications for the Best Paper Award. In 2017, 73 papers were published by COAP. The recipient of the 2017 Best Paper Award is Sergio González-Andrade of Escuela Politécnica Nacional de Ecuador for his paper "A preconditioned descent algorithm for variational inequalities of the second kind involving the p-Laplacian operator" published in volume 66, pages 123–162. This article highlights the research related to the award winning paper.

In the awarded paper [2], the author proposes a preconditioned descent algorithm in function spaces for the numerical solution of a class of variational inequalities of the second kind involving the p-Laplacian operator, which arise as necessary optimality condition of the nonsmooth energy minimization problem:

$$\min_{W_0^{1,p}(\Omega)} J(u) := \frac{1}{p}\int_\Omega |\nabla u|^p dx + g\int_\Omega |\nabla u|\, dx \; - \int_\Omega f u\, dx, \qquad (\mathcal{P})$$

where $1 < p < \infty$, $g > 0$ and $f \in L^{p'}(\Omega)$.

The motivation for studying this class of variational inequalities lies on the fact that they can be used to model the flow of a particular class of viscoplastic materials, the so-called Herschel–Bulkley fluids. The main feature of this model is the use of a parameter p, known as the flow index, which measures the degree to which the fluid is shear-thinning ($1 < p < 2$), shear-thickening ($p > 2$) or behave as a Bingham fluid ($p = 2$). [1,4].

For the case $1 < p < 2$, the algorithm is proposed in a tailored Hilbert function space, in which the preconditioner is defined and the existence of admissible search directions is proved. This approach starts by introducing the spaces $H_0^u(\Omega)$, which are defined as the completion of $\mathcal{D}(\Omega)$, the space of smooth functions compactly supported in Ω, with respect to the norm

$$\|z\|_{H_0^u} = \left(\int_\Omega (\epsilon + |\nabla u|)^{p-2} |\nabla z|^2\, dx\right)^{\frac{1}{2}},$$

for $u \in W_0^{1,p}(\Omega)$ and $\epsilon > 0$, and satisfy the embeddings $H_0^1(\Omega) \subset H_0^u(\Omega) \subset W_0^{1,p}(\Omega)$. Given an iterate $u_k \in W_0^{1,p}(\Omega)$, the descent direction $w_k \in H_0^{u_k}(\Omega) \subset W_0^{1,p}(\Omega)$ is obtained by solving the variational equation

$$\int_\Omega (\epsilon + |\nabla u_k|)^{p-2}(\nabla w_k, \nabla v)\, dx = -J'_\gamma(u_k)v, \forall v \in H_0^{u_k}(\Omega),$$

where J_γ is a Huber regularized version of the objective functional. Existence of a unique solution for this equation is obtained by direct application of the Riesz–Fréchet representation theorem. Further, the proof of convergence of the descent algorithm is achieved by verifying the Zoutendijk condition in the corresponding Hilbert function spaces.

For the case $p \geq 2$, this approach is no longer suitable, since, given an elliptic preconditioner P_k, the unique solution for the variational equation

$$P_k(w_k, v) = -J'_\gamma(u_k)v, \ \forall v \in W_0^{1,p}(\Omega)$$

satisfies $w_k \in W_0^{1,p'}(\Omega) \supset W_0^{1,p}(\Omega)$. This fact prevents the author from directly constructing a descent algorithm, since there is no guarantee that the update u_{k+1} lies in $W_0^{1,p}(\Omega)$, but only in $W_0^{1,p'}(\Omega)$. To overcome this difficulty, an alternative algorithm in a finite element space is proposed and a global convergence result is stated and proved.

Although descent methods are usually slow, the design of suitable preconditioners and the use of an appropriate line search algorithm, based on interpolation of the objective functional, may lead to fast converging algorithms, which only require the solution of one linear system per iteration. Further, since the algorithms are proposed in function spaces, mesh independence may be expected. It is worth mentioning that although the paper is concerned with the numerical simulation of complex fluids, the approach may also be applied to similar problems arising in image processing or elasticity.

The ideas developed in this paper have motivated the research group to face new challenges in the field. For instance, earlier this year, in [3], the proposed algorithm was successfully used as a smoothing-optimization algorithm in a multigrid-optimization (MG/OPT) method developed for the numerical simulation of the three classical models for viscoplastic materials: Bingham, Herschel–Bulkley and Casson. Finally, in the current days, the ideas in the awarded paper [2] are being further developed for vectorial variational inequalities of the second kind posed in free-divergence Sobolev spaces. The main objective is to numerically simulate nonlinear p-Stokes systems and the Herschel–Bulkley model.

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Sergio González-Andrade received his Ph.D. in Applied Mathematics from Escuela Politécnica Nacional de Ecuador in 2008. From 2009 to 2010, he worked as a pos-doctoral researcher at the Karl-Franzens University of Graz, within the SFB Research Center on Mathematical Optimization and Applications in Biomedical Sciences. Since 2010 he is Associate Professor at Escuela Politécnica Nacional de Ecuador, and since 2013 he coordinates the Modelling and Simulation Area at the Ecuadorian Research Center on Mathematical Modelling—MODEMAT (www.modemat.epn.edu.ec). His research areas include the numerical simulation of complex fluids, optimization and variational methods for nonsmooth problems, and multigrid methods.