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COAP 2012 Best Paper Prize

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Each year, the editorial board of Computational Optimization and Applications (COAP) selects a paper from the preceding year's publications for the Best Paper Award. In 2012, 133 papers were published by COAP. The recipients of the 2012 Best Paper Award are Chungen Shen of Shanghai Finance University, Sven Leyffer of Argonne National Laboratory, and Roger Fletcher of the University of Dundee for their paper "A nonmonotone filter method for nonlinear optimization" published in volume 52 pages 583–607. This article highlights the research related to the award winning paper.

The paper describes a new nonmonotone globalization strategy for nonlinearly constrained optimization. In particular, the paper describes a trust-region sequential quadratic programming (SQP) method and introduces a nonmonotone filter method that promotes global convergence from remote starting points. The use of a nonmonotone filter leads to a proof of fast local convergence near regular minimizers without the need for second-order correction steps.

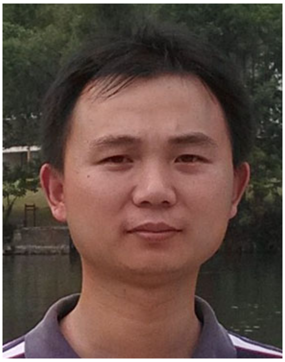
Filter methods [1] have been proposed as an alternative to penalty function methods. A filter is a list of pairs of constraint violation and objective function values such that no pair dominates any other pair. A filter method accepts a new iterate if it improves the constraint violation or objective value for all filter pairs. When filter methods were first introduced, it was conjectured that these methods did not suffer from the Maratos effect [3], which prevents Newton steps from being accepted arbitrarily close to a solution, and therefore may prevent fast local convergence. Filter methods can be shown to converge superlinearly for the standard Maratos example. Unfortunately, Nick Gould and Philippe Toint constructed an example where the Newton step increases both the objective function and the constraint violation arbitrarily close to a solution, which demonstrated that filter methods too suffer from the Maratos effect. As a result, several authors have proposed and analyzed second-order correction steps [5], or suggested filter methods based on the Lagrangian function [4] to show fast local convergence.

The paper introduces a filter method that employs two filters: a global (g-) filter that safeguards convergence towards stationary points, and a local (l-) filter that takes over when Newton-steps are detected. The l-filter is a nonmonotone filter that accepts new iterates that are not dominated by more than $M \geq 1$ filter entries ($M = 0$ corresponds to the standard monotone filter). The use of an l-filter allows the algorithm to ignore old and outdated filter entries, and the nonmonotonicity leads to a proof of fast local convergence. The algorithm switches from the g-filter to the l-filter whenever the trust-region becomes inactive at the solution of the QP. It switches back to the g-filter (flushing the entries from the l-filter) if a step is not acceptable by the l-filter.

The global convergence analysis for the nonmonotone filter is a straightforward extension of the analysis for standard filter methods [2]. The filter essentially acts as a mechanism that drives iterates towards the feasible set. Once iterates are sufficiently feasible, the method enforces convergence using a standard (unconstrained) sufficient reduction condition. For the nonmonotone filter, the method employs a nonmonotone sufficient reduction condition.

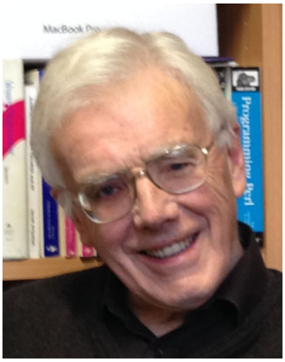
The transition to fast local convergence is assured by switching to the (initially empty) nonmonotone l-filter. The convergence analysis makes use of a penalty function, though this function is never formed or used in the algorithm. The contours of an ℓ_1 exact penalty function are straight lines in the filter diagram, and the nonmonotonicity assumption ensures that every second QP step can be shown to make progress (by interpreting it as a second-order correction step).

Much of this work was carried out while Chungen was visiting Argonne. He started to look at a nonmonotone filter version and realized that outdated filter entries may still prevent fast local convergence. This motivated the use of the l-filter and the development of suitable switching condition between the local and global filter. An interesting open question is whether it might be possible to rely on just a single nonmonotone filter.



Chungen Chen is an associate professor of the Department of Applied Mathematics at Shanghai Finance University, China. He received his Ph.D. from Tongji University in 2010. His research interests focus on numerical optimization and its applications in finance.

Sven Leyffer is a Senior Computational Mathematician at Argonne National Laboratory. He received his Ph.D. from the university of Dundee in 1994. He is interested in the development of methods for large-scale nonlinear optimization, mixed-integer optimization, and optimization problems with complementarity constraints. Sven is a SIAM Fellow.



Roger Fletcher is an Emeritus Professor and Senior Research Fellow at the University of Dundee. He received his Ph.D. from Leeds University, and previously held positions at Leeds (1963–1969), AERE Harwell (1969–1973), and the University of Dundee (1973–2005). Roger was elected a Fellow of the Royal Society at Edinburgh (1988), and of the Royal Society (2003). He won the Dantzig Prize in 1997 and together with Sven Leyffer and Philippe Toint the SIAM/MOS Lagrange Prize in 2006. Roger was awarded the Royal Medal of the Royal Society at Edinburgh in 2008. He is a SIAM Fellow and a Fellow of the IMA.

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