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# COAP 2011 Best Paper Award

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In each year, the Computational Optimization and Applications (COAP) editorial board selects a paper from the preceding year's publications for the "Best Paper Award." The recipients of the award for papers published in 2011 are Ian Kopacka of the Austrian Agency for Health and Food Safety and Michael Hintermüller of Humboldt-University of Berlin for their paper "A smooth penalty approach and a nonlinear multigrid algorithm for elliptic MPECs" published in Volume 50 pages 111–145. Among the 81 papers published by COAP in 2011, the editor board would also like to recognize the following two papers for honorable mention: "Corrector-predictor methods for sufficient linear complementarity problems" by Filiz Gurtuna, Cosmin Petra, Florian A. Potra, Olena Shevchenko and Adrian Vancea published in Volume 48, pages 453–485 and "A framework for analyzing sub-optimal performance of local search algorithms" by Alexander G. Nikolaev, Sheldon H. Jacobson, Shane N. Hall and Darrall Henderson published in Volume 49, pages 407–433. In this article, we highlight the research of Kopacka and Hintermüller related to their award winning paper.

Optimal control problems for variational inequalities have been a subject of interest in the optimal control community of distributed parameter systems starting from the 1980s. However, only rather recently this problem class was connected to mathematical programs with equilibrium constraints (MPECs) in function space. In this respect, [1], an earlier work of this year's best paper awardees, was perhaps the first contribution which related the well-established terminologies such as weak, $C$-, or strong stationarity of a solution of an MPEC to control problems in function space. In fact, not unexpectedly entirely new stationarity conditions, pertinent to function space settings only, could be revealed. For instance, the new notion of $\mathcal{E}$-almost $C$-stationarity was derived upon an application of Egorov's Theorem. Further it was found that for the particular problem class of optimal control of the obstacle problem, i.e.

$$\begin{aligned} &\text{minimize} \quad J(y,u) \quad \text{over } H_0^1(\Omega)\times U \\ &\text{subject to} \quad y\in \operatorname{argmin}\left\{\frac{1}{2}\|\nabla y\|^2_{L^2(\Omega)} - \int_\Omega (f+u)y\,dx : y\geq 0 \text{ a.e. in } \Omega\right\}, \end{aligned}$$

where $J$ denotes a smooth objective, e.g. the tracking-type objective, $U = L^2(\Omega)$, $f \in L^2(\Omega)$ is given and $\Omega$ is a bounded and sufficiently regular domain; the seminal work by Fulbert Mignot and Jean-Pierre Puel [7] provided a version of strong stationarity, a concept of course unknown to those authors at the time when their paper was published.

Unfortunately, [7] could not resolve the issue of so-called (pointwise) upper level constraints on the control variable $u$ ($y$ is the state). This was primarily due to the fact that properties of the directional derivative of the control-to-state mapping and a regularity gain of the control variable could be exploited strategically in the derivation of strong stationarity. Such a regularity gain of the control, however, cannot be expected when pointwise upper level control constraints are present and the derivation of stationarity principles remained elusive.

This year's award paper [2] closes this analytical gap by establishing a version of $C$-stationarity for the optimal control of the obstacle problem when $u$ is required to be in a convex and closed proper subset $U$ of $L^2(\Omega)$. This not only opens up new avenues in the stationarity analysis of MPECs in function space—and quite some research has been sparked already in this direction—but it also influences the design of solution algorithms.

For the efficient numerical solution of the resulting first order optimality system two types of multigrid schemes are proposed in [2]: The first one being related to Achi Brandt's *Full Approximation Scheme*, a very effective, but analytically less understood variant of multigrid methods; and the second one being related to the multigrid method of the second kind as conceived by Wolfgang Hackbusch for other problem classes. The latter scheme has the advantage of a rigorous convergence analysis, but the resulting algorithm is somewhat more expensive. It should be noted that multigrid methods for the numerical solution of a class of MPECs have hardly been studied before.

As indicated above, this research, which is part of Ian Kopacka's PhD thesis, sparked further investigations of optimal control problems for variational inequalities or MPECs in function space in Michael Hintermüller's research group, but recently also in other groups. Earlier this year, set-valued analysis tools could be extended to treating, e.g., pointwise constraints on the gradient of the state in the lower level problem (rather than uni- or bilateral constraints on the state as in the obstacle case) but without upper level constraints. In this vein, in the recent work [4] second-order Mosco epi-derivatives are utilized in a non-polyhedric constraint setting in order to obtain a rather sharp stationarity principle for such problems. Upper level constraints are considered in the recent preprint [3], where Mordukhovich's limiting variational calculus and co-derivatives are employed and compared against three other approaches. These analytical investigations led to the algorithmic development in [5], where a bundle-free implicit programming type method is proposed and analyzed. One of the key ingredients is a structurally smoothed directional derivative of the control-to-state mapping in the critical case of biactivities. It can be shown that the overall method is guaranteed to converge to an $\mathcal{E}$-almost $C$-stationary point and even to B(ouligand) stationary points under conditions.

Currently, in Michael Hintermüller's research group, the above developments are being further pursued for sharpening the analytical tools and for solver development in the context of function space problems; moreover, new tools are being used for analyzing specific applications such as calibration problems in mathematical finance or problems in phase separation; see [6] for the latter.

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