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COAP 2010 Best Paper Award

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In each year, the Computational Optimization and Applications (COAP) editorial board selects a paper from the preceding year's COAP publications for the "Best Paper Award." The recipients of the award for papers published in 2010 are Marco D'Apuzzo, Valentina De Simone, and Daniela di Serafino at the Second University of Naples for their paper entitled "On mutual impact of numerical linear algebra and large-scale optimization with focus on interior point methods" published in Volume 45, pages 283–310. Valentina De Simone and Daniela di Serafino wish to dedicate this award to Marco D'Apuzzo, who passed away suddenly on June 24, 2009, at the age of 46. Marco was a clever and enthusiast researcher as well as a close friend, and Daniela and Valentina would have really liked to share with him their joy for this success, of which he would have been certainly proud.

The paper is the result of work done by the authors at the Second University of Naples during the years 2004–2007, as a part of national research projects on large-scale nonlinear optimization, funded by the Italian Ministry for University and Research with Gianni Di Pillo as Principal Investigator. A goal of the authors' activity was the development of an Interior Point code based on Potential Reduction methods for solving large-scale convex quadratic programming problems. From the beginning it was clear to the authors that an effective implementation of Interior Point methods is highly dependent on the availability of effective linear algebra algorithms and software for the solution of the KKT linear systems that provide the directions used to update the approximations of the optimal solution. These systems have the following saddle-point form:

$$\begin{pmatrix} H+C & -J^T \\ -J & -D \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix},$$

where H is the Hessian of the Lagrangian of the optimization problem, J is the Jacobian of the constraints, and C and D are semidefinite diagonal matrices that account for the logarithmic barrier functions of the Interior Point methods. When dealing with large-scale problems, the cost of solving KKT systems via direct methods may become prohibitive in terms of both memory and time requirements, so the authors focused on the use of iterative solvers. In this case, the application of preconditioning techniques is mandatory, due to the inherent ill conditioning of the KKT linear systems that increases as the Interior Point iterate progresses towards the optimal solution.

Starting from the work in [3, 8–10, 12], the authors devoted their attention to Constraint Preconditioners, that appeared very promising for the problems under considerations. A very interesting property of these preconditioners is that they allow the use of the Conjugate Gradient method even though the KKT systems are not positive definite. The authors analyzed the behavior of a Constraint Preconditioner coupled with the Conjugate Gradient method in the context of a Potential Reduction method for quadratic programming, and showed the efficiency of this iterative approach with respect to the application of direct solvers [4]. The use of iterative techniques led the authors to the convergence analysis of an inexact infeasible Potential Reduction method, from which they concluded that the residual of the KKT system must satisfy a bound depending on the duality gap [6]. This, in turn, suggested an adaptive stopping criterion for the inner iterations that relates the accuracy in the solution of the KKT system to the quality of the current iterate. This avoids the expense of achieving too much accuracy when the iterates are far from the optimal solution, and enables better computational efficiency [5]. Further work was devoted to devising a strategy for choosing when to recompute the preconditioner or reuse the previously computed one, with the final aim of reducing the overall Potential Reduction time [7].

During their work, the authors soon realized that the previous aspects are common to Interior Point methods for nonlinear optimization. Therefore, they decided to review them in a more general framework. On the other hand, the iterative solution of KKT systems received a lot of attention while the authors were working on this subject, and the excellent survey in [2], that goes into numerical techniques for the solution of saddle-point systems in a wider context than Optimization, was dated 2005, thus not including the most recent advances. Finally, the authors desired to analyze in more detail how linear algebra and optimization have influenced and stimulated each other in recent years, to provide a sequel to the interesting essay in [11]. This led the authors to their COAP 2010 paper, where they discuss the mutual impact of recent developments in numerical linear algebra and optimization, focusing on Interior Point methods for the solution of large-scale nonlinear optimization problems and on iterative linear algebra techniques. To make the presentation self-consistent, they show how KKT systems arise in the Interior Point framework (as well as in the Sequential Quadratic Programming one) and report the main properties of the KKT matrix. Then they discuss the solution of large and sparse KKT linear systems, addressing in particular three fundamental issues: preconditioning, with special attention to Constraint Preconditioners, adaptive stopping criteria for the inner iterations, and control of the inertia of the KKT matrix. The last aspect is important since the inertia reveals if the optimization problem is locally strictly convex at the current iterate. Finally, the authors report some experiences with their Potential Reduction software.

Current research activities of Valentina De Simone and Daniela di Serafino still include preconditioning techniques for optimization problems. They are interested in preconditioner updates for sequences of linear systems arising in trust-region and regularized subproblems in the context of nonlinear least-squares [1], as well as in algebraic multilevel preconditioners for the optimality systems associated with elliptic PDE-constrained optimization problems.

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