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COAP 2004 Best Paper Award

In each year, the Computational Optimization and Applications (COAP) editorial board selects a paper from the preceding year's COAP publications for the "Best Paper Award". The recipients of the award for papers published in 2004 are Luca Bergamaschi, University of Padova, Italy, Jacek Gondzio, University of Edinburgh, Scotland, and Giovanni Zilli, University of Padova, Italy, for their paper "Preconditioning Indefinite Systems in Interior Point Methods for Optimization", published in Volume 28, pages 149–171. This paper describes a class of indefinite preconditioners for reduced KKT systems arising in quadratic and nonlinear optimization with interior point methods. Spectral analysis of preconditioners is given and the improvements resulting from the use of primal-dual regularization are demonstrated. Computational results are reported for the application of the preconditioner to a variety of medium-size convex quadratic programming problems.

Work on the paper began during the Summer of 2001, about a year after the authors had first met when Zilli invited Gondzio to give a series of lectures on interior point methods in Padova in September, 2000. The authors had complementary research backgrounds: Bergamaschi and Zilli worked on the theory and implementation of iterative methods for linear and nonlinear systems of equations [2], while Gondzio worked on the design and implementation of interior point methods for large-scale optimization [3]. This was essential since together, the three authors could tackle in depth a problem which required expertice in several areas. At that time Bergamaschi and Zilli were impressed with the recent development of Lukšan and Vlček [6], who applied conjugate gradients to indefinite systems, and with the later analysis of Keller, Gould and Wathen [4] of indefinite constraint preconditioners. They convinced Gondzio to look into the issue. In the meantime, Rozlozník and Simoncini [7] came up with an improved understanding of issues connected with indefinite preconditioning (in the context of saddle point problems arising in partial differential equation); these developments led the authors to take a closer look at the problem.

Starting in July, 2001, the authors began to incorporate into Gondzio's interior point code HOPDM (higher order primal dual methods) four iterative techniques: conjugate gradients, BiCGstab, GMRES and QMR. A description of these schemes can be found in Kelley's book [5] and the references therein. The authors realized that the key difficulty in the solution of augmented systems of Newton equations in interior point methods for quadratic and nonlinear programming (otherwise known as reduced KKT systems) was due to the loss of sparsity caused by the presence of the Hessian matrix Q in the system

$$\begin{bmatrix} -(Q+\Theta_P^{-1}) & A^T \\ A & \Theta_D \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}. \tag{1}$$

Here $Q \in \mathcal{R}^{n\times n}$ is the Hessian of the Lagrangian, $A \in \mathcal{R}^{m\times n}$ is the constaint Jacobian, and $\Theta_P \in \mathcal{R}^{n\times n}$ and $\Theta_D \in \mathcal{R}^{m\times m}$ are very ill-conditioned diagonal scaling matrices resulting from the logarithmic barrier terms in interior point methods. This observation motivated a choice of preconditioner which attempts to produce a more sparse (1,1) block in system (1). A radical solution in which the block $-(Q + \Theta_P^{-1})$ is replaced by $-\text{diag}(Q + \Theta_P^{-1})$ turned out to have several appealing features. After initial encouraging computational results obtained in November 2001, the authors looked closer into the advantages resulting from the use of primal-dual regularization [1]: The regularized indefinite system displays better conditioning than the original system and it is easier to precondition. Indeed, the spectral analysis of the original and regularized systems performed in the early months of 2002 shed light on this issue. The authors met again in April 2002 and completed a draft version of the paper soon afterwards. The paper was submitted to COAP in July 2002.

Bergamaschi and Zilli continue to work on iterative methods for linear and nonlinear systems of equations, while Gondzio is collaborating with Grothey on direct methods for structured optimization problems; they recently solved a quadratic programming problem with $353 \cdot 10^6$ constraints and over 10^9 variables. Together, the authors are investigating iterative methods for interior point methods in (nonconvex) nonlinear optimization.

References

1. A. Altman and J. Gondzio, "Regularized symmetric indefinite systems in interior point methods for linear and quadratic optimization," Optimization Methods and Software, vol. 11/12, pp. 275–302, 1999.
2. L. Bergamaschi and G. Zilli, "Parallel inexact Newton and interior point method," Ann. Univ. Ferrara—Sez VII—Sc. Mat., XLV 2000, pp. 467–478.
3. J. Gondzio, "HOPDM (version 2.12)—a fast LP solver based on a primal-dual interior point method," European Journal of Operational Research, vol. 85, pp. 221–225, 1995.
4. C. Keller, N.I.M. Gould, and A.J. Wathen, "Constraint preconditioning for indefinite linear systems," SIAM Journal on Matrix Analysis and Applications, vol. 21, pp. 1300–1317, 2000.
5. C.T. Kelley, "Iterative methods for linear and nonlinear equations," vol. 16 of Frontiers in Applied Mathematics, SIAM, Philadelphia, 1995.
6. L. Lukšan and J. Vlček, "Indefinitely preconditioned inexact Newton method for large sparse equality constrained nonlinear programming problems," Numerical Linear Algebra with Applications, vol. 5, pp. 219–247, 1998.
7. M. Rozložník and V. Simoncini, "Krylov subspace methods for saddle point problems with indefinite preconditioning," SIAM Journal of Matrix Analysis and Applications, vol. 24, pp. 368–391, 2002.

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Jacek Gondzio received his MSc in Automatic Control and Computer Science in 1983, and his Ph.D. in Automatic Control and Robotics in 1989, both from the Department of Electronics, Warsaw University of Technology, Poland. Since 1983 he has been a research fellow in the Polish Academy of Sciences. He spent 15 months in Paris and 5 years in Geneva as a postdoctoral research fellow. In 1998 he moved to the School of Mathematics at the University of Edinburgh, where he was a lecturer (1998–2000), then a reader (2000–2005), and a Professor of Optimization since 2005.

Giovanni Zilli received his degree in Mathematics in 1967 from the University of Bologna, Italy. In 1969 he moved to the University of Padova, as an Assistant Professor of Mathematical Analysis. Since 1985 he has been an Associate Professor of Numerical Calculus at the Faculty of Engineering of the University of Padova.