The Rise and Fall of Armies*

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WORKING PAPER This Version: January 17, 2017

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Abstract

For a thousand years, income growth was associated with a rising military employment share. But this share peaked in the early 20th century, after which military employment shares fell with income growth. I argue that rising military shares were driven by structural change out of agriculture, and the recent declines are driven by substitution from soldiers towards military goods. I document evidence for this substitution effect: as countries' incomes rise, the ratio of their military expenditure share to their military employment share rises too. I introduce a game theoretic model of growth and warfare that reproduces the time series patterns of military expenditure and employment. The model also correctly predicts the cross-sectional pattern, that military employment and expenditure shares are decreasing in income during wars. Finally, I show that faster economic growth can reduce military expenditure in the long run.

JEL-Codes: E10, F51, H56, O40

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*I am grateful for helpful comments and guidance from Michelle Garfinkel, Mikhail Golosov, Jon Hamilton, Erik Hurst, Loukas Karabarbounis, Robert Lucas, Brent Neiman, Nancy Qian, James Robinson, and Richard Romano. I am also grateful for feedback from seminar participants at the University of Florida and working group participants at the University of Chicago. Shubhi Agarwal provided excellent research assistance.

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1 Introduction

> Today, machines and technology are permitting economy of manpower on the battlefield, as indeed they are in the factory. But the future offers even more possibilities for economy. I am confident the American people expect this country to take full advantage of its technology - to welcome and applaud the developments that will replace wherever possible the man with the machine.
>
> - Gen. William Westmoreland (1969)

Over one thousand years, military employment has exhibited a hump-shaped pattern. The military employment share of populations rose steady as countries became richer, peaking in the early 20th century, and falling over the past hundred years. This paper presents a theory that explains this long-run rise and decline of armies. I conclude that this pattern is driven by two macroeconomic factors: transition out of agriculture and substitution from military labor towards military goods. This substitution effect explains the continually rising ratio of military expenditure shares to employment shares, which I document across countries. Corroborating the model, the theory correctly predicts that the correlations between income and employment and expenditure shares are negative in the cross-section among opposing countries, when controlling for per capita income.

The fundamental driver of the time series patterns is income growth. When countries are poor, they must spend most of their resources in agriculture to feed the population (Allen, 2000). As incomes grow, countries are able to spend more income on everything else, including warfare: military employment and expenditure shares both rise. Military personnel and equipment are substitutes in the production of military power[1], so as productivity rises, countries shift workers to the goods producing sector, raising the military expenditure to employment ratio. In the long

[1]A large literature estimates that capital and labor are substitutes, or have become substitutes, in the second half of the 20th century in most countries, e.g. Karabarbounis and Neiman (2013). Within the military sector, DeBoer and Blackley (1990) estimate the elasticity of substitution to be greater than one but declining using US data since 1929. Clark (1978) estimates that capital and labor are substitutes in the US Navy using data after the Korean War. However, this conclusion is not universal; Ridge and Smith (1991) estimate an elasticity of substitution of one in UK data.

run, the military employment share asymptotes to zero, while the expenditure share stabilizes.

Figure 1 plots the time series for the military employment to population ratio for an unbalanced panel of 12 European countries over 1,000 years. The data are presented in 25-year bins, using estimates on historical army sizes from Sorokin (1937) and more recent military data from the National Military Capacity Database v4.0.[2]. The time series documents a 900 year rise, followed by a 100 year decline in the military employment share.

Figure 1: Military and Rural Employment Shares

Figure 1 also plots the time series for the rural population share of the 12 countries over 1,000 years. The data are estimates from Bairoch, Batou, and Chevre (1988) and Bairoch (1991). Countries are predominantly rural and agrarian for centuries. As they develop, they urbanize and shift out of agriculture and into other sectors.[3] As a share of their income, they spend more on everything nonagricultural, including warfare.

Countries have also seen the ratio of their military expenditure share to employment share rise regularly over the past century. For example, Figure 2 plots France's ratio since 1914 CE. If

[2]The 4th version is an update of the original Singer, Bremer, and Stuckey (1972)

[3]A large literature on structural change examines this pattern and its relationship with economic growth. See for example Kuznets (1966), Maddison (1980), Baumol, Blackman, and Wolff (1985), and Herrendorf, Rogerson, and Valentinyi (2013) among many others.

soldiers' wages relative to the price of military equipment rise with income, then an increasing ratio of expenditure to employment shares reflect substitution from soldiers towards equipment. This is the central economic force driving the long run decline in military employment in the model.

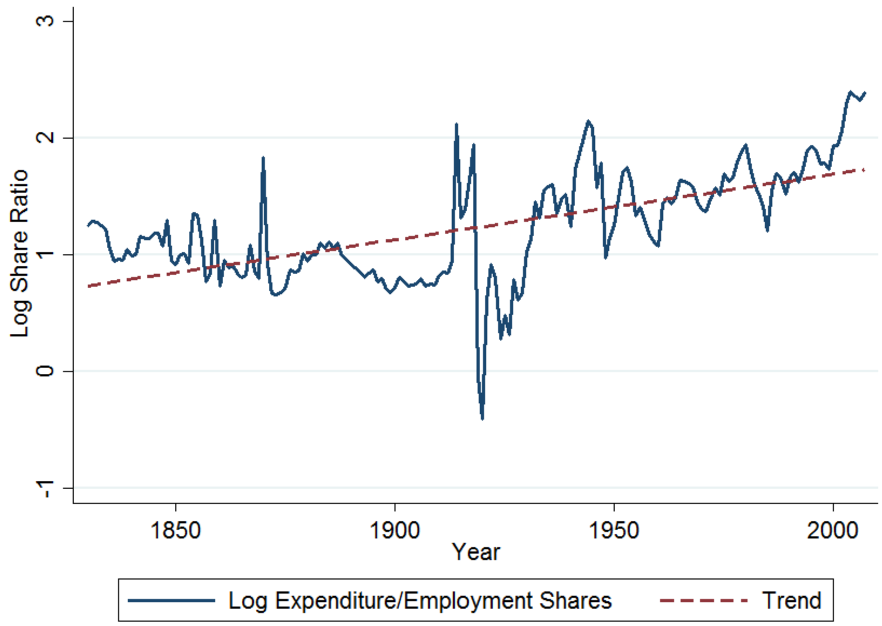

Figure 2: French Military Substitution

Recent research has made considerable progress on understanding the economic forces behind conflict and warfare. This paper falls in two of these literatures. First, it contribute to the theoretical literature describing the relationship between economic factors and conflict in general equilibrium (e.g. Haavelmo (1954), Hirshleifer (1988), Grossman (1991), Skaperdas (1992), and Powell (1993) among many others.). Second, it joins a literature analyzing economic determinants of conflict from a macroeconomic perspective (e.g. Grossman and Kim (1996), Gonzalez (2007), Acemoglu, Golosov, Tsyvinski, and Yared (2012) and Garfinkel, Skaperdas, and Syropoulos (2015)). Third, it contributes to a literature researching economic determinants and consequences of very long run trends in conflict and warfare (e.g. Findlay and O'Rourke (2007), Dincecco and Prado (2012), Arbatli, Ashraf, and Galor (2015)). Finally, it joins the broader literature of economic determinants of conflict, which Garfinkel and Skaperdas (2007), Anderton and Carter (2007), and Blattman and Miguel (2010) survey.

The theory says nothing about why countries go to war. Rather, it assumes that war occurs, and makes conditional predictions about countries choices of employment and expenditure. Therefore it omits some common ingredients that affect countries' propensity to battle. There is no role for bargaining or transfers between countries, which in rational models (e.g. De Mesquita (1985)) can often prevent war. The baseline model is not dynamic, which in some cases admits war as an equilibrium strategy - as in Fearon (1995) or Garfinkel and Skaperdas (2000) - although the dynamic extension to the model implies that forward-looking countries can reduce or eliminate war in equilibrium. Decision makers make choices in the best interest of their countries, which is not generally realistic; Jackson and Morelli (2007) consider how the political bias of decision makers affects their decisions to lead a country into war, and how this provide incentives for countries to choose biased leaders to gain bargaining position. There is no uncertainty over the state of game or imperfect information, which can increase the prevalence of war, as in Brito and Intriligator (1985).

The remainder of this paper is organized as follows: Section 2 examines the empirical patterns, Section 3 describes the baseline model, Section 4 analyzes asymmetric equilibria, Section 5 considers a dynamic extension to the model and its implications for the long run, and Section 6 concludes.

2 Stylized Facts

In this section I describe the data sources and three stylized facts: (1) the military employment share of the population rises, peaks, and falls with income; (2) the ratios of military expenditure shares to employment shares are increasing in income; (3) countries' military employment and expenditure shares are decreasing in their income relative to their opponents'.

The military employment share of the population rises then falls with income. Figure 1 plotted this pattern over a millennium for 12 large European countries. But the nonmonotonic relationship with income is more general. To test this relationship I run the following regression:

$$\log s_{j,t} = \beta_0 + \beta_1 \log(Y_{j,t}) + \beta_2 \log(Y_{j,t})^2 + \alpha_j + \varepsilon_{j,t} \tag{1}$$

where $s_{j,t}$ is the military employment share of country j in year t, $Y_{j,t}$ is their real gdp per capita, and α_j is a country fixed effect. Table 1 reports the results in an unbalanced panel from 1816-2007.

The rise and fall of armies is evident in the baseline specification, which is reported in column (1): The coefficient on $\log(Y_{j,t})$ is positive and the coefficient on the quadratic term is negative, confirming the hump-shaped pattern. Countries choose different military employment shares in wartime versus peacetime. It's possible that the unconditional trends could be driven by the declining frequency of warfare. So I also run the regressions only including observations for which a country was at war, which is when the theory applies. Column (2) shows these results for the quadratic regressions, and confirms the theory during war years. Indeed, during war years the trend is stronger, with a larger coefficient on the linear term and a more negative coefficient on the quadratic term. Lastly, to see whether recently declining employment shares are robust to omission of the world wars, columns (3) and (4) omit the quadratic term and show that employment shares decline with income since 1950.

Military employment shares rise and fall, but the evidence for the long run trend of military expenditure shares is mixed. I run regression (1) for expenditure shares and the baseline estimate is reported in column (5) of table 1, which does not estimate a negative coefficient on the quadratic term. Yet, the regression using only war observations (column (6)) does find a negative coefficient, so the data is not clear on the long run trend. The baseline model described in Section 3 features a military expenditure share that is nonzero in the limit.

Dependent variable:	Log Employment Share				Log Expenditure Share		Log Share Ratio	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log(GDP Per Capita)	1.405^{***}	6.655^{***}	-0.111^{***}	-0.285^{***}	-1.431^{***}	4.816^{***}	0.146^{***}	0.242^{**}
	(0.112)	(0.815)	(0.015)	(0.088)	(0.158)	(1.013)	(0.015)	(0.096)
Log(GDP Per Capita)^2	-0.081^{***}	-0.393^{***}			0.098^{***}	-0.273^{***}		
	(0.007)	(0.049)			(0.010)	(0.061)		
Constant	-11.057^{***}	-32.492^{***}	-4.619^{***}	-3.283^{***}	0.424	-23.455^{***}	-0.529^{***}	0.581
	(0.447)	(3.339)	(0.116)	(0.688)	(0.633)	(4.068)	(0.169)	(1.157)
Sample years	All	War only	All (≥1950)	War only(≥1950)	All	War only	All	War only
Observations	11,436	616	7,211	276	10,655	568	10,401	558

standard errors in parentheses *p<0.1; **p<0.05; ***p<0.01

Table 1: Military Shares and Income

The ratio of a country's military expenditure share of GDP to its military employment share of the population increases with income. Figure 2 plotted this pattern for France, but it is evident across countries. To demonstrate, I regress the log ratio $\log(R_{j,t})$ for country j in year t on log GDP per capita $\log(Y_{j,t})$:

$$\log R_{j,t} = \beta_0 + \beta_1 \log(Y_{j,t}) + \alpha_j + \varepsilon_{j,t}$$

where α_j are country fixed effects. I report estimates of β_1 in column (7) in Table 1. The baseline regression is statistically significant at the 1% level and implies a 0.14 elasticity of the share ratio with respect to income. Column (8) reports the regression using only war observations, which is only significant at the 5% level but has a larger estimate than the baseline. These estimates correspond to an elasticity of substitution for military labor and goods between 1.15 and 1.24 when taken to the model.

A country's military employment and expenditure share is decreasing in its income relative to its opponents. This is true unconditionally, but it is the conditional pattern that is relevant to the theory. Conditional on a country's own per capita income, its military employment and expenditure shares are also decreasing in their relative income. Specifically, the model predicts that the shares are decreasing in the relative *aggregate* income, while per capita income is what drives the time series patterns. To test this effect, I regress log employment or expenditure shares $\log(s_{j,t})$ on the log of the ratio $\log(\tilde{Y}_{j,t})$ of country j's GDP relative to its opponents[4] in year t, and a quadratic function of log GDP per capita $\log(Y_{j,t})$:

$$\log R_{j,t} = \beta_0 + \beta_1 \log(\tilde{Y}_{j,t})_t + \beta_2 \log(Y_{j,t}) + \beta_3 \log(Y_{j,t})^2 + \varepsilon_{j,t}$$

I report the results in Table 2. In all cases, the shares are decreasing in relative income. When controlling for per capita income, the elasticity is between -0.10 and -0.14 and is statistically significant. Section 4 describes why this is predicted in wars between asymmetric countries.

Conditioning on per capita income is crucial. Skaperdas and Syropoulos (1997) show in a general setting that more productive combatants will retain a lower share of their surplus,

[4]When wars occur between more than two parties, relative income is the total income of a country's opponents divided by the number of its allies, including itself.

Dependent variable:	Log Employment Share			Log Expenditure Share		
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Relative GDP)	-0.0509***	-0.138***	-0.105***	-0.0223	-0.139***	-0.140***
	(0.0191)	(0.0215)	(0.0220)	(0.0219)	(0.0233)	(0.0245)
Log(GDP Per Capita)		0.418***	4.030***		0.597***	0.475
		(0.0551)	(0.686)		(0.0600)	(0.786)
Log(GDP Per Capita)^2			-0.231***			0.00772
			(0.0437)			(0.0497)
Constant	-4.447***	-7.623***	-21.57***	-2.956***	-7.541***	-7.067**
	(0.0469)	(0.421)	(2.672)	(0.0536)	(0.463)	(3.086)
Observations	589	589	589	545	545	545

Standard errors in parentheses
* $p < .1$, ** $p < .05$, *** $p < .01$

Table 2: Military Shares and Relative Income

because returns to military power are decreasing and countries only capture a fraction of available surplus. Unconditional evidence is not strong; in Table 2, the coefficients on relative income in the unconditional regressions are close to zero. But incentives are not the whole story: productivity growth affect the supply of military inputs. As countries grow, laborers move out of agriculture and become available for military service, and as countries substitute from soldiers to goods, the growth rate of military power catches up to the growth rate of income. Conditioning on per capita income controls for these effects and supports the asymmetric predictions of the theory.

2.1 Data Sources

Military expenditure and employment data in the post-Napoleonic era are from the National Military Capacity Database v4.0[5] which covers up to 146 countries in an unbalanced panel from 1816-2007. Expenditure data from 1914 is nominal US dollars, which I convert to real expenditures using the NIPA GDP deflator. Before 1914, expenditure data is nominal British pounds, which I convert to real expenditures using the Bank of England's historical GDP deflators and historical USD/GBP exchange rates. The employment data include only troops

[5] The 4th version is an update of the original Singer, Bremer, and Stuckey (1972) dataset through 2007, and with more complete coverage. One drawback of this dataset is that expenditures include compensation to soldiers inconsistently; in some cases it's excluded entirely, in others it is partial, and in others it is fully included. This makes trends in military goods expenditures difficult to estimate. Yet, the share ratio rises with income whether or not soldier compensation is included, if soldier compensation does not grow sufficiently faster than income. This is because the soldier share of the population decreases with income.

under command of the national government which are intended for combat with foreign parties, excluding national reserves, civil defense units, and forces of feudal lords not operating with the central government. It excludes non-military support staff that are contracted by the military.

Historical military employment data before 1816 are from Sorokin (1937), which includes estimates of historical army sizes for 12 European countries. This dataset is not perfectly comparable to the National Military Capacity's dataset, chiefly because the Sorokin dataset only measures army sizes during wartime. Accordingly, I adjust the Sorokin estimates by a constant factor, so that average employment shares are equal for both sources during the common years 1816-1925 in which both sources have data.

Conflict data are from the Correlates of War database (Sarkees and Wayman, 2010), which catalogs the participants and years for all conflicts classified as wars between states from 1816-2007. Lastly, income and population data are from The Maddison Project (2013).

3 Model

In the baseline model, two countries are endowed with productivity and population and always go to war. They solve a static game, which has a Nash equilibrium in pure strategies.

3.1 Preferences and Technology

There are two countries, indexed by $j \in 1, 2$. Country j has population N_j and productivity Z_j, which are exogenous[6]. Each country's decisions are made by a government, which maximizes the consumption C_j of its citizens.

Citizens serve one of two roles. They can be soldiers or they can be workers. $N_{X,j}$ denotes the number of soldiers and $N_{Y,j}$ denotes the number of workers, which must add up to the population:

$$N_{X,j} + N_{Y,j} = N_j \tag{2}$$

[6]Exogenous productivity is chosen for simplicity and to understand implications for military allocations conditional on aggregate income. But it is not an innocuous assumption in a model with conflict; Gonzalez (2005) shows that the possibility of appropriation can reduce the incentive to adopt new technologies.

Workers have productivity Z_j and produce three types of goods: agriculture A_j, guns G_j, and surplus Y_j. The resource constraint for goods is:

$$Z_j N_{Y,j} = A_j + G_j + Y_j \tag{3}$$

Agriculture is used to feed the population. Each person consumes ν units of agriculture, so the agricultural goods constraint is:

$$A_j = \nu N_j \tag{4}$$

As a result, ν behaves like a subsistence constraint.

Guns and soldiers are used to produce military power, X_j, with a production function $f(N_X, G)$. Assume this production function is quasi-convex and has constant returns to scale:

$$X_j = f(N_{X,j}, G_j) \tag{5}$$

The substitutability of guns and soldiers in this production function will be crucial for determining the long-run behavior of the economy.

3.2 War

The two countries go to war, where military power is used to compete over a share θ of the joint surplus of the two countries. Without loss of generality, label the countries 1 and 2. The contestable share of the joint surplus of the two countries is $\theta(Y_1 + Y_2)$. The war function $\Gamma(X_1, X_2)$ determines the share of this surplus that accrues to country 1. $\Gamma(\cdot, \cdot)$ is increasing in the first argument, decreasing in the second argument, bounded by $\Gamma(\cdot, \cdot) \in [0, 1]$, and is symmetric so that $\Gamma(X_1, X_2) = 1 - \Gamma(X_2, X_1)$.

A country uses its surplus for consumption, net of war gains or losses. Then the consumption for country 1 is determined by:

$$C_1 = \Gamma(X_1, X_2)\theta(Y_1 + Y_2) + (1 - \theta)Y_1 \tag{6}$$

The country faces a trade-off in turning goods into consumption. Spending more on Y_1 increases the total surplus, but decreases the resources that can be spent on X_1, reducing the share of total surplus that is retained. When $\theta = 1$, all output is potentially contestable, and war resembles the structure in Skaperdas (1992).

3.3 Equilibrium Definition

A Nash equilibrium in this economy consists of labor allocations $N_{X,j}, N_{Y,j}$; goods allocations A_j, G_j, Y_j, C_j; and military powers X_j; given populations and productivities Z_j, N_j, such that each country $j \in 1, 2$:

1. Satisfies goods constraints (3) and (4)
2. Satisfies its population constraint (2)
3. Satisfies military constraints (5) and (6)
4. Maximizes its consumption as the best response to the other country's choices X_i, Y_i, $i \neq j$

3.4 Equilibrium Conditions

The country's decision can be rewritten as an unconstrained maximization problem in two variables - guns and soldiers - here expressed from the perspective of country 1:

$$\max_{N_{X,1}, G_1} (\Gamma(f(N_{X,1}, G_1), X_2)\theta + 1 - \theta)(Z_1(N_1 - N_{X,1}) - \nu N_1 - G_1) + \Gamma(f(N_{X,1}, G_1), X_2)\theta Y_2$$

The optimality conditions of this maximization problem are lengthy, but can be written in an intuitive form by substituting in some definitions. I also omit for readability the arguments of the function Γ, its partial derivative with respect to the first argument Γ_1, and marginal return to military from increasing soldiers f_N and guns f_G. The equilibrium condition for guns is

$$\Gamma_1 f_G \theta (Y_1 + Y_2) = \Gamma\theta + 1 - \theta \tag{7}$$

On the left hand side is the marginal increase in consumption, from an increase in military power, due to an increase in guns. On the right hand side is the marginal consumption given up by shifting output from surplus to guns. The equilibrium condition for soldiers is

$$\Gamma_1 f_N \theta (Y_1 + Y_2) = (\Gamma\theta + 1 - \theta) Z_1 \tag{8}$$

On the left hand side is the marginal increase in consumption from increased military power, and on the right hand side is the marginal consumption lost from shifting soldier to workers. This marginal cost has a coefficient of Z_1, the productivity level, which differs from the guns equilibrium condition. This is because at the margin, using output for guns reduces surplus one for one, but employing workers as soldiers reduces surplus by the factor Z_1.

Dividing equation (8) by equation (7) shows that the marginal product of guns in the military power function must with productivity relative to the marginal product of soldiers:

$$\frac{f_N}{f_G} = Z_1 \tag{9}$$

Productivity Z_1 is the relative price of soldiers with respect to guns. If guns and soldiers are substitutes, then rising productivity will increase guns more than one for one, relative to soldiers.

3.5 Functional Forms

Some functional forms are useful to further characterize the baseline economy. The production function for military power is a CES aggregator of guns and soldiers:

$$f(N_X, G) \equiv \left(\alpha N_{X,j}^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) G_j^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{10}$$

The elasticity of substitution ϵ will be a crucial parameter determining the relationship between income growth and warfare.

Lastly, a functional form is needed for the war function $\Gamma(X_1, X_2)$, satisfying the conditions in section 3.2. One such function is that countries receive a share of surplus equal to their share of military power:

$$\Gamma(X_1, X_2) = \frac{X_1}{X_1 + X_2} \tag{11}$$

This has the convenient property of homogeneity of degree zero, and is the linear form of the Tullock (1980) rent-seeking function, which is often used in the conflict literature (Garfinkel and Skaperdas, 2007).

3.6 Symmetric Equilibrium

I first examine symmetric equilibria, where $Z_1 = Z_2$ and $N_1 = N_2$. Section 4 considers the implications of asymmetric equilibria.

With the assumed functional forms, the equilibrium condition (7) becomes

$$(1-\alpha)\frac{c}{g} = \frac{\theta + 2(1-\theta)}{\theta}(\frac{x}{g})^{\frac{\epsilon-1}{\epsilon}} \tag{12}$$

where lower case letters denote per capita variables, e.g. $c \equiv \frac{C}{N}$. Substituting in $x = f(n_X, g)$, equation (12) becomes

$$(1-\alpha)\frac{c}{g} = \frac{2-\theta}{\theta}\left(\alpha(\frac{n_X}{g})^{\frac{\epsilon-1}{\epsilon}} + 1 - \alpha\right) \tag{13}$$

and substituting for the marginal products, equation (9) becomes

$$\frac{\alpha}{1-\alpha}(\frac{g}{n_X})^{\frac{1}{\epsilon}} = Z \tag{14}$$

Then given productivity Z, the symmetric equilibrium is characterized by three variables (n_X, g, c) and three equations: (12), (14), and the per capita symmetric budget constraint,

$$Z(1-n_X) = \nu + g + c \tag{15}$$

3.7 Equilibrium in the Limit

To understand how productivity growth affects the economy, it is useful to start by considering equilibrium in the limit as Z becomes large. Define shares of total income $s_C \equiv \frac{c}{Z}$ and $s_G \equiv \frac{g}{Z}$. Shares s_C, s_G, and n_X are all bounded by 0 and 1. Let $(\overline{s_C}, \overline{s_G}, \overline{n_X})$ denote their limits, if they exist.

Express the condition (14) in terms of shares and rearrange to get

$$\frac{s_G}{n_X} = Z^{\epsilon-1}(\frac{1-\alpha}{\alpha})^{\epsilon} \tag{16}$$

If $\epsilon > 1$, then the right hand side of this equation grows as productivity Z grows. On the left hand side, s_G is positive and bounded above by one, so as Z becomes large, it must be that n_X becomes small: $\overline{n_X} = 0$. This is why it central that guns and soldiers are substitutes to imply a declining share of soldiers as income grows. Guns become cheap relative to soldiers as productivity grows. If the two inputs are substitutes in the military function, then this price change implies a quantity shift from soldiers towards guns. Equation (16) maps directly to the regressions estimated in columns (5) and (6) of Table 1. This corresponds to an elasticity of substitution ϵ between 1.15 and 1.24.

Given that $\overline{n_X} = 0$, condition (13) implies $\overline{s_C} = \overline{s_G}\frac{2-\theta}{\theta}$ in the limit. And the budget constraint (15) becomes $1 - n_X = \frac{\nu}{Z} + s_G + s_C$, so in the limit as $Z \to \infty$,

$$1 = \overline{s_G} + \overline{s_C} \tag{17}$$

This also implies that as productivity rises, the share of total income spent on agriculture, $\frac{\nu N}{ZN}$ goes to zero. Solving for each share yields $\overline{s_C} = 1 - \frac{\theta}{2}$ and $\overline{s_G} = \frac{\theta}{2}$.

Figure 3 plots the symmetric equilibrium for various levels of productivity, to show how the equilibrium evolves as productivity grows from $Z = 1$ to high levels. In this example $\nu = .9$, so initially agriculture is 90% of total income Z. As income grows and a lower share needs to be paid to agriculture, more is spent on consumption, which rises to its' long run share $\overline{s_C}$, which in this example is .5 because $\theta = 1$. Spending on both military inputs rise as a share of income initially as the economy transitions out of agriculture. But eventually the substitution effect starts to dominate, so the soldier share falls to $\overline{n_X} = 0$ while the guns share rises to $\overline{s_G}$.

4 Asymmetric Equilibrium

In this section, I relax the assumption of symmetry to explain the third stylized fact: countries' military employment and expenditure shares are decreasing in their income relative to their opponents'.

Countries' incentives are quantitatively different when they are richer or poorer than their neighbors. When conditioning on their productivity level, a country's military employment

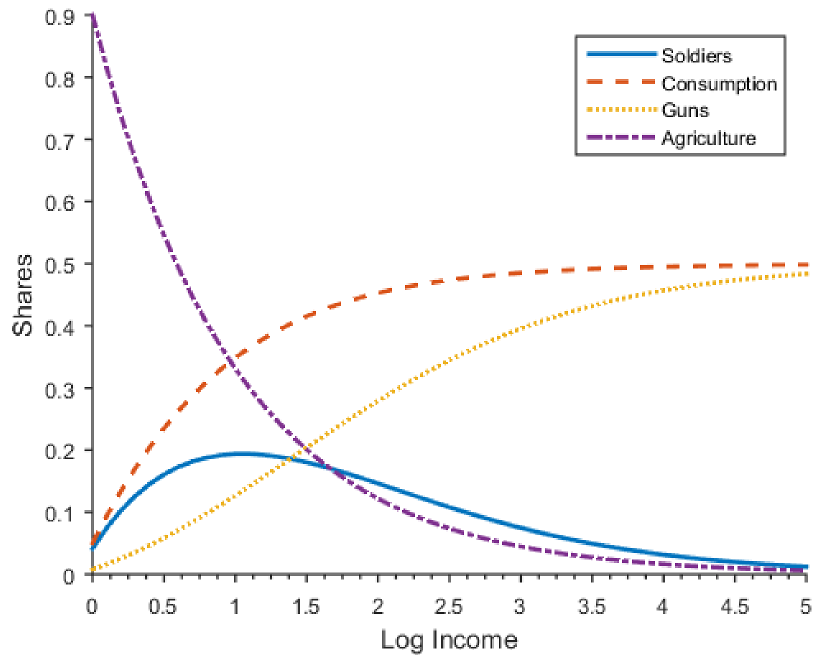

Figure 3: Example Symmetric Equilibria as Income Grows

and expenditure share is decreasing in its income relative to its opponents. The game-theoretic nature of the model generates cross-sectional implications that differ observably from the time series implications. In the time series, the model implies predicts that the employment share rises and falls with income and the expenditure share rises and asymptotes. Whereas in the cross section when controlling for productivity, the model predicts that both the employment share and expenditure share are decreasing in relative income.

When a country's income rise in the asymmetric case, then the country's incentive to spend on war falls. The available surplus rises less than one for one with the country's income, because it only retain a fraction of the total available surplus, yet the marginal share retained decreases more than one for one, for a given guns expenditure share share. A lower guns share is required to increase the marginal share retained, so guns expenditure shares are decreasing in neighbors' incomes. The optimal allocation of workers implies that solider are proportional to guns by a factor that depends on the level of productivity. So conditional on the productivity level, employment shares are also decreasing in relative income.

To characterize some properties of asymmetric equilibria, I approximate the model in the long run when incomes are large. In general, the asymmetric equilibrium must be calculated numerically, but I can analytically characterize asymmetric behavior for the approximation. The first implication of the long run approximation is that for large G, $f(N_X, G) \approx (1-\alpha)^{\frac{\epsilon}{\epsilon-1}} G$. The second implication is for large Z, $\frac{\nu}{Z} \approx 0$. Then the equilibrium condition for guns (7) becomes

$$\frac{G_2}{(G_1+G_2)^2}(Z_1N_1 - G_1 + Z_2N_2 - G_2) = \frac{G_1}{G_1+G_2} + \frac{1-\theta}{\theta} \tag{18}$$

Rewrite the guns expenditures as shares of total income $G_1 = s_{G,1}Z_1N_1$, and define relative aggregate income of country 1 $z_1 \equiv \frac{Z_1N_1}{Z_2N_2}$, to get an expression for country 1's best response function $g(s_{G,2}, z_1)$ to country 2's guns share and relative income:

$$s_{G,1} = g(s_{G,2}, z_1) = \frac{s_{G,2}}{z_1}\left(\sqrt{\theta\frac{z_1+1}{s_{G,2}}} - 1\right) \tag{19}$$

The asymmetric Nash Equilibrium satisfies two equations: $s_{G,1} = g(s_{G,2}, z_2; \theta)$ and $s_{G,2} = g(s_{G,1}, \frac{1}{z_2}; \theta)$. For z_2 near 1, this solution is close to $(s_{G,1}, s_{G,2}) = (\frac{1}{\theta}, \frac{1}{\theta})$, the symmetric outcome from Section 3.3. But when incomes are asymmetric, the guns shares diverge.

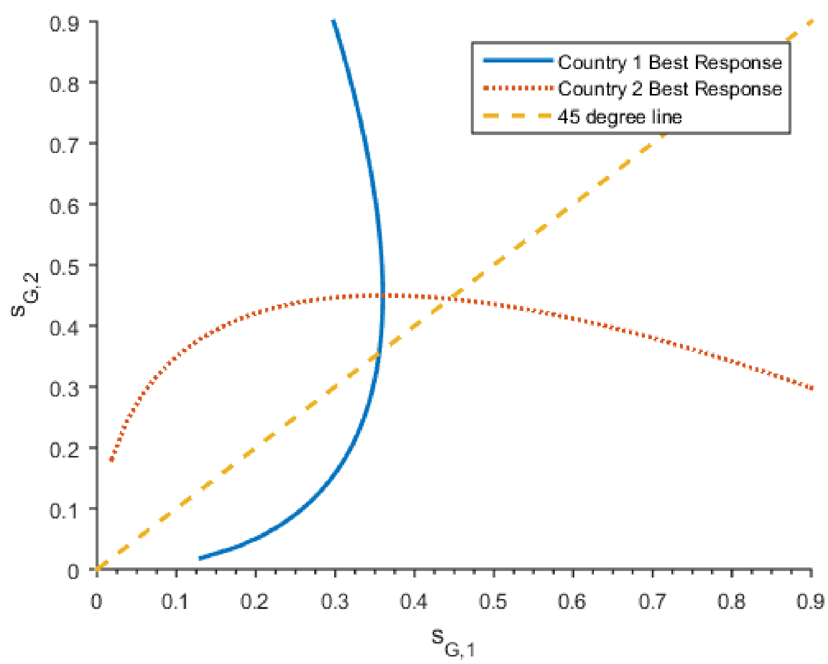

Figure 4: Asymmetric Best Response Functions

The guns share best response is decreasing in relative income z_1. When country 1 is richer, country 1 chooses a lower guns share for any given foreign guns share. When $z_1 > 1$ (i.e.

country 1 is richer), then $s_{G,1} < s_{G,2}$. This case is plotted in Figure 4, for $\theta = .8$ and a 20% income difference. The best response lines cross above the 45 degree line, so the richer country chooses a lower guns share in equilibrium. Figure 5 plots the equilibrium guns share of country 1 against its relative income $\frac{Z_1 N_1}{Z_2 N_2}$ for several values of θ. As country 1 becomes richer relative to country 2, it chooses a lower guns share.

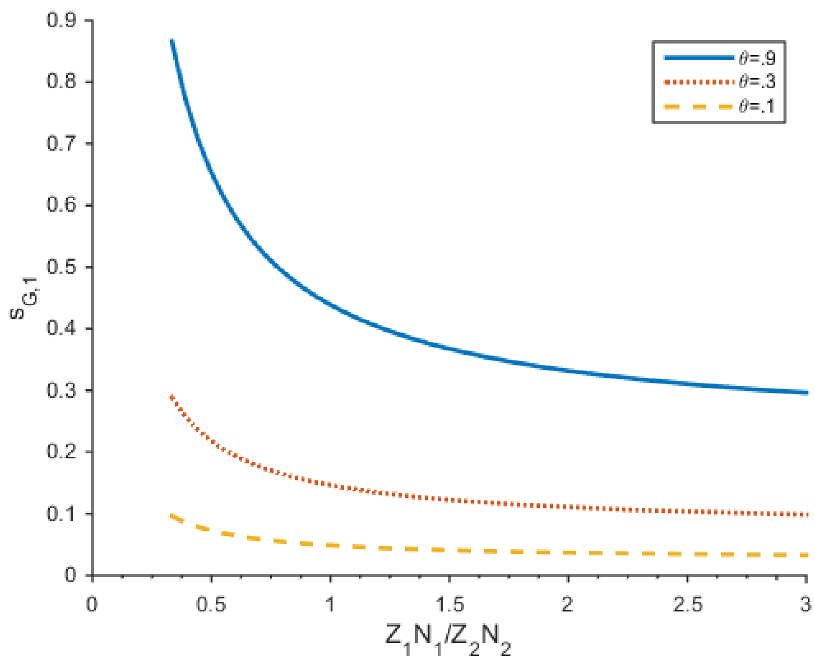

Figure 5: Equilibrium Guns Expenditure Shares and Relative Incomes

Military employment shares must be decreasing in relative income too. The military employment share is always related to the guns share by equation (16), so that for the shares are always proportional for a given productivity level. Foreign endowments do not enter this relationship. So if relative populations change, or if the foreign country becomes more productive, the employment share will change one for one with the guns share. If domestic productivity increases, this reduces the guns share, but reduces the employment share more than one for one, because the substitution effect shifts soldiers into production. In both cases, the military employment share is decreasing in relative income.

5 The Repeated Game

The baseline model in Section 3 considers a static Nash equilibrium, which is a good approximation when wars are infrequent, so that time discounting is significant between periods. In this section, I generalize this setup to a repeated game environment. I show that when the discount factor is sufficiently small, the baseline results hold: military employment shares rise and fall, and countries substitute from soldiers to guns. As discount factors or productivity growth rise in the repeated game, the long run military expenditure share falls. When discount factors or productivity growth are sufficiently high, countries can play a strategy that eliminates warfare entirely. I begin by considering grim trigger strategies which are easily characterizable, and follow by considering punishment strategies that are subgame perfect.

Each country's decisions are made by a government, which now maximizes the present discounted utility of its citizens. The period utility function is power utility with parameter σ over consumption per capita, so the present discounted utility in period $t = 0$ is

$$\sum_{t=0}^{\infty} \beta^t (\frac{C_{j,t}}{N_{j,t}})^\sigma \tag{20}$$

When $\beta = 0$, the country's objective is the same as in the baseline model.

When $\beta > 0$, countries can support a better equilibrium than the static Nash equilibrium. In general, repeated games admit many possible equilibria. First consider grim trigger strategies: countries play one allocation as long as the other country does the same, but if one country deviates from this allocation, the other country plays the grim trigger allocation forever after. Within this set of strategies, the best sustainable Nash equilibrium is achieved by choosing the minmax punishment: the allocation that gives the other country the least utility, given that the other country is playing its best response. In this game, the minmax allocation is the one that maximizes military power $f(N_X, G)$. So country j's minmax allocation solves the following maximization problem:

$$\max_{N_{X,j}, G_j} f(N_{X,j}, G_j)$$

$$\text{s.t. } Z_j(1 - N_{X,j}) = \nu N_{X,j} + G_j$$

which shares an optimality condition with the baseline model: $\frac{\alpha}{1-\alpha}(\frac{G_j}{N_{X,j}})^{\frac{1}{\epsilon}} = Z_j$.

An equilibrium is sustainable when playing a grim trigger strategy if countries prefer the equilibrium in perpetuity, over the best one period deviation followed by the best response to the minmax strategy. This implies the following condition, where C_t denote period t consumption in a sustainable equilibrium, $\hat{C}_t$ denotes the best one period deviation, and C_t^{MM} denotes the best response to the minmax strategy:

$$\sum_{t=0}^{\infty} \beta^t (\frac{C_t}{N_t})^{\sigma} \geq \hat{C}_0 + \sum_{t=1}^{\infty} \beta^t (\frac{C_t^{MM}}{N_t})^{\sigma} \tag{21}$$

The best sustainable grim trigger equilibrium is the one where (21) holds with equality.

Better equilibria can be sustained when countries are more patient, or when wars are less frequent. When $\beta = 0$, the best sustainable equilibrium must also be the best one period response, yielding the static Nash equilibrium. To see how the equilibrium changes when β increases, consider condition (21) in the limit when expenditure shares are constant. Let s_C denote the sustainable limiting consumption share, let $\hat{s}_C$ denote the best one period deviation consumption share, and let s_C^{MM} denote the best response consumption share to the minmax. The best sustainable grim trigger equilibrium in the limit satisfies:

$$s_C^{\sigma} \sum_{t=0}^{\infty} \beta^t Z_t^{\sigma} = (Z_0 \hat{s}_{C,0})^{\sigma} + (s_C^{MM})^{\sigma} \sum_{t=1}^{\infty} \beta^t Z_t^{\sigma} \tag{22}$$

When productivity growth is constant at rate μ, equation simplifies to:

$$s_C^{\sigma} = \hat{s}_{C,0}^{\sigma}(1 - \beta(1+\mu)^{\sigma}) + (s_C^{MM})^{\sigma} \beta(1+\mu)^{\sigma} \tag{23}$$

The best grim trigger equilibrium is a weighted average of the best response and the minmax response. When $\beta(1+\mu)^{\sigma}$ is higher, more weight is placed on the punishment, allowing for a better achievable best response $\hat{s}_{C,0}^{\sigma}$ and a better sustainable equilibrium.

Faster productivity growth μ also improves the sustainable equilibrium. With high productivity growth, countries act as if they are more patient, knowing that more consumption will be lost if they are punished by the grim trigger. Either higher patience or faster growth improve the long run outcome by reducing the guns share and increasing the consumption share. Furthermore, they improve the equilibrium at every point in transition. Figure 6 plots the transition path for grim trigger equilibria for several values of β. All cases exhibit the rise and fall of armies. When $\beta = 0$, the best equilibrium is static Nash. As countries become more patient, better equilibria are sustainable, and the soldier share falls at every level of income.

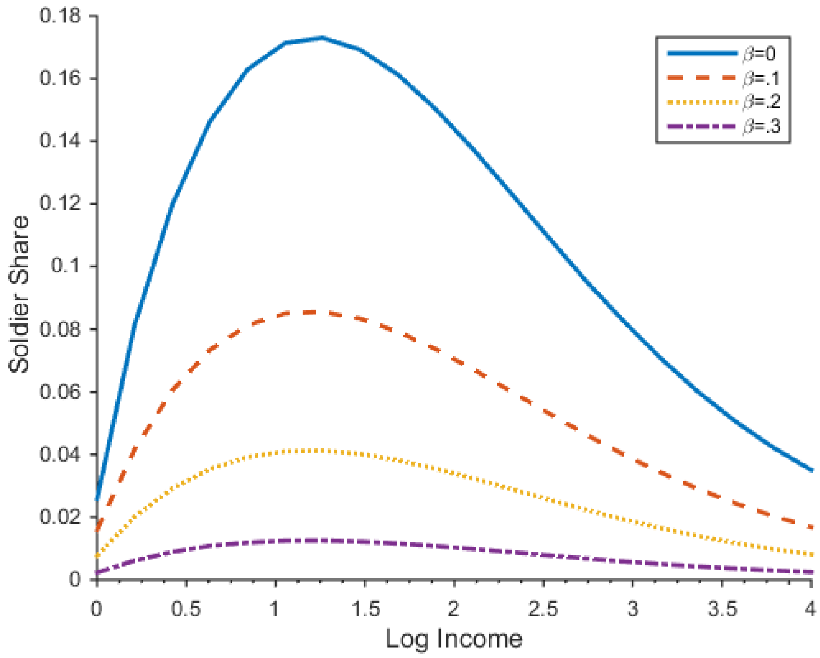

Figure 6: Soldier Shares for Various Grim Trigger Equilibria

Higher discount factors or economic growth rates result in lower long run guns expenditure shares. In the baseline model, the contestable share of surpluses θ controlled the long run guns share. When dynamics are introduced, this ingredient is unnecessary to match empirical expenditure shares, which are much lower than $\frac{1}{2}$. The grim trigger equilibria in Figure 7 all have $\theta = 1$. When $\beta = 0$ and the economy is in the static Nash equilibrium, the guns share converges to $\frac{1}{2}$. As β increases, the long run guns share decreases.

Sufficiently high patience or economic growth can eliminate war entirely. Grim trigger strategies do not generally produce the best possible equilibrium, nor are they subgame perfect, so I next consider sustainable subgame perfect equilibria with punishment, as in Abreu (1988). Countries play a strategy where, were either country to deviate from the sustainable allocation, both countries play the worst possible allocation for k periods, then revert to the sustainable

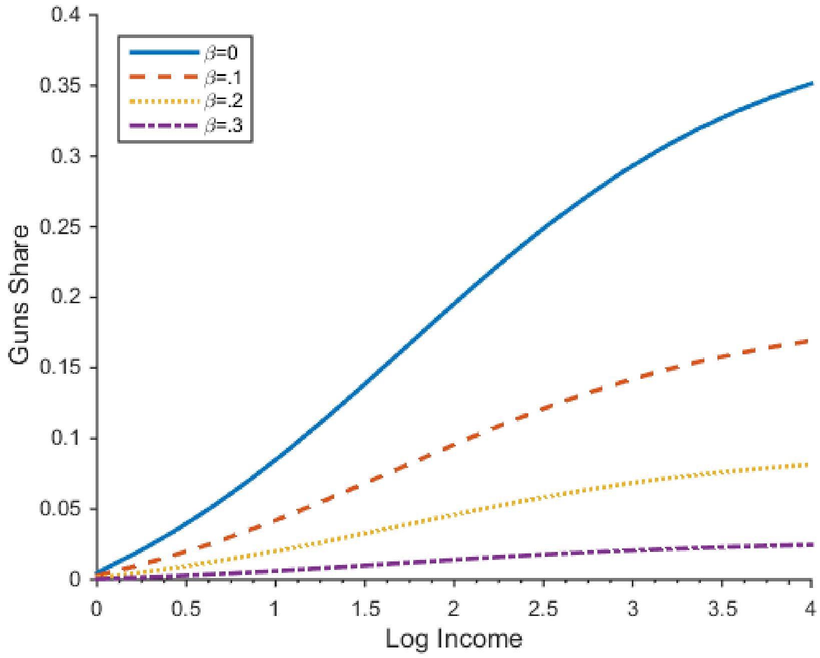

Figure 7: Guns Shares for Various Grim Trigger Equilibria

allocation.

This equilibrium has two dynamic conditions. Countries must prefer not to deviate from the sustainable allocation:

$$\sum_{t=0}^{\infty} \beta^t (\frac{C_t}{N_t})^\sigma \geq \hat{C}_0 + \sum_{t=1}^{k} \beta^t (\frac{C_t^P}{N_t})^\sigma + \sum_{t=k+1}^{\infty} \beta^t (\frac{C_t}{N_t})^\sigma \tag{24}$$

where C_t^P denotes the punishment level of consumption in period t. The punishment must also be credible for the equilibrium to be subgame perfect, so that a country prefers to accept its punishment for k periods, rather than deviating:

$$\sum_{t=0}^{k-1} \beta^t (\frac{C_t^P}{N_t})^\sigma + \sum_{t=k}^{\infty} \beta^t (\frac{C_t}{N_t})^\sigma \geq \hat{C}_0^P + \sum_{t=1}^{k} \beta^t (\frac{C_t^P}{N_t})^\sigma + \sum_{t=k+1}^{\infty} \beta^t (\frac{C_t}{N_t})^\sigma \tag{25}$$

where $\hat{C}_t^P$ denotes the one period best response to the punishment level of consumption in period t.

The structure of the game has to be modified to allow for an equilibrium without war. Assume that when both countries choose zero guns $G_t = 0$, then the country spends its entire income on agriculture and consumption, so that $Z_t N_t = C_t$. Better equilibria are supported by worse punishments, so assume the worst possible punishment: $C_t^P = 0$. When supporting no war, and with productivity growth rate μ, condition (24) becomes:

$$Z_0 \sum_{t=0}^{k} \beta^t (1+\mu)^{\sigma t} \geq \hat{C}_0 \tag{26}$$

k periods of no war must be preferable to one period of deviation. Condition (25) becomes:

$$Z_0 \beta^k (1+\mu)^{\sigma k} \geq \hat{C}_0^P \tag{27}$$

which implies that one period of deviation form punishment must not be worth delaying a return to no war for an additional period. If there is some k such that these conditions are satisfied, no war is sustainable.

The conditions to sustain no war are characterizable analytically in the long run symmetric case, where the set of feasible punishment lengths k is increasing in the discount factor β and the growth rate μ:

Theorem 1 *If the two countries have the same productivities and populations, and $\theta = 1$, then an equilibrium without war exists and is subgame perfect in the long run where $\nu = 0$ and $X = (1-\alpha)^{\frac{\epsilon}{\epsilon+1}} G$, if there exists some punishment length k such that $\sum_{t=0}^{k} \beta^t (1+\mu)^{\sigma t} \geq 2^\sigma$ and $\beta^k (1+\mu)^{\sigma k} \geq (\sqrt{2}-1)^{2\sigma}$.*

Proof. The two countries are symmetric, so the best one period deviation to the no war allocation is $\hat{C}_t = 2Z_t N_t$, because one country can choose an arbitrarily tiny military power greater than zero, and capture the entire surplus.

In the long run where $\nu = 0$ and $X = (1-\alpha)^{\frac{\epsilon}{\epsilon+1}} G$, and when $\theta = 1$, country 1's best response condition for guns (7) becomes

$$\frac{G_1}{(G_1+G_2)^2}(2Z - G_1 - G_2) = \frac{G_2}{G_1+G_2} \tag{28}$$

so the best one period deviation to the punishment $G = Z$ has guns choice $\hat{G}^P$ that satisfies

$$\frac{\hat{G}^P}{(\hat{G}^P + Z)^2}(Z - \hat{G}^P) = \frac{Z}{\hat{G}^P + Z}$$

Applying the quadratic formula and taking the unique feasible solution, the best one period deviation to the punishment is given by

$$\hat{G}^P = Z(\sqrt{2}-1) \tag{29}$$

And the associated consumption $\hat{C}^P = \Gamma(\hat{G}^P, Z)(Z - \hat{G}^P)$ is

$$\hat{C}^P = Z(\sqrt{2} - 1)^2 \tag{30}$$

With these responses $\hat{C}$ and $\hat{C}^P$ determined, the optimality condition (26) becomes

$$\sum_{t=0}^{k} \beta^t (1+\mu)^{\sigma t} \geq 2^\sigma \tag{31}$$

and the credibility condition (27) becomes

$$\beta^k (1+\mu)^{\sigma k} \geq (\sqrt{2} - 1)^{2\sigma} \tag{32}$$

■

Faster productivity growth and higher patience increase the set of feasible strategies that sustain a subgame perfect equilibrium without war. The lefthand sides of both the optimality condition (31) and the credibility condition (32) are increasing in $\beta(1+\mu)^\sigma$. Figure 8 plots the lower bound of feasible values of $\beta(1+\mu)^\sigma$ for both conditions, for a variety of punishment lengths. When $\beta(1+\mu)^\sigma$ is above both lower bounds, no war is sustainable for the given punishment length k. If $\beta(1+\mu)^\sigma$ is sufficiently small, then these conditions cannot be satisfied, and war will occur.

6 Concluding Remarks

This paper has presented a theory of warfare and economic growth that explains the very long run rise and decline in the military employment share. This pattern is driven by income growth, which transitions workers out of agriculture, then incentivizes countries to substitute from military employment towards military equipment. The model correctly predicts that the correlations between income and employment and expenditure shares are negative in the cross-section among opposing countries, when controlling for per capita income. When the model is dynamic, faster income growth can reduce military expenditure and employment, and potentially eliminate war entirely.

Long run macroeconomic changes affect incentives and costs of war. This paper shows that macroeconomic determinants of sectoral composition change warfare over time. With

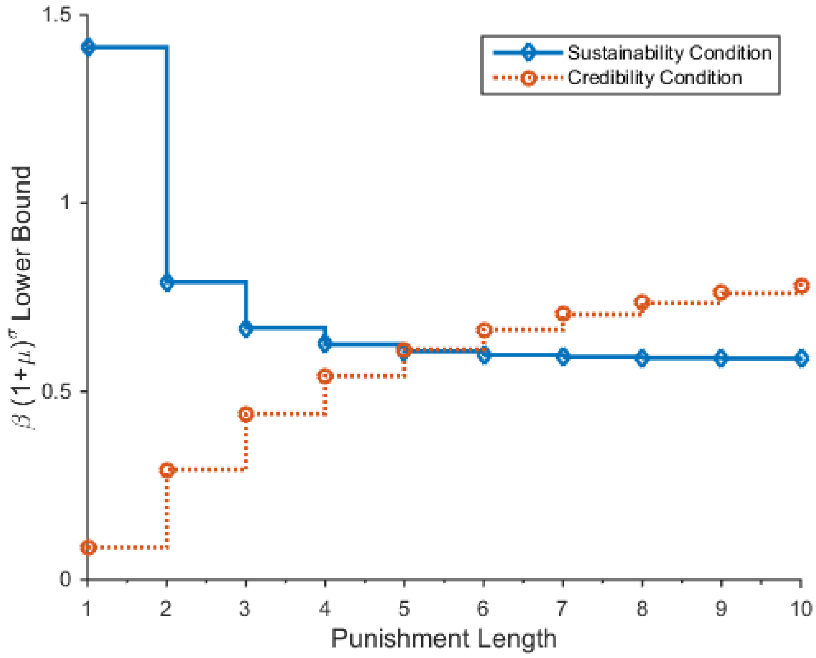

Figure 8: Parameter Lower Bounds Supporting Equilibrium Conditions Without War

continuing productivity growth, war will look very different than our historical experiences. As the nature of conflict evolves with the macroeconomy, important research questions arise. How do these trends affect the dynamics of war and peace? Are static theories of conflict affected by macroeconomic changes? Are the dynamics theories of macroeconomic growth affected by warfare? These are fruitful questions for future research.

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