Moderating Capital Bubbles Under Dispersed Information

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Motivation

- Information frictions can amplify business cycles
- When agents can build capital: excess volatility of investment $\implies$ *capital bubbles*
- What can be done?
  - One possible tool: asset market policy
  - In 2020 central banks started directly buying private capital (ETFs and corporate bonds)

Contribution

- Study a business cycle model with dispersed information featuring capital and learning from endogenous signals
- Specifically interested in effects of sentiment shocks, propagated by capital accumulation
- Find that sentiments amplify business cycles when information is incomplete
- Examine policy interventions using public information to moderate the inefficient volatility
- A beneficial policy: raise future asset returns during recession to encourage investment

Incomplete Information in Macroeconomics

- Modern business cycle models with incomplete information; large literature following Woodford (2003). In these papers, noise is either exogenous or short-lived. Attenuation of real shocks is typical (e.g. Angeletos and La'O 2010)
- Models usually have no capital + endogenous information. Adams (2021) derives method to solve this combination
- Some recent examples of amplification of real shocks: Venkateswaran 2014, Angeletos and Lian 2020, Chahrour and Gaballo 2020
- Other amplification examples: non-fundamental shocks (e.g. Lucas 1972, Lorenzoni 2009), relaxing RE (Greenwood and Hanson 2015, Gaibaix 2016), partial equilibrium (large finance literature)

Model Structure

- Agents live on Lucas-style “islands” where they work and own capital
- Islands have different information sets, produce differentiated goods with island-specific inputs
- Islands face productivity shocks, with aggregate and idiosyncratic components
- Islands receive news about future productivity, but with error: the “sentiment”
- Agents know some things about the macroeconomy, but not everything, so must make inference from limited information $\Longrightarrow$ inefficient outcomes

Model: Household Preferences

- Unit measure of islands $\mathcal{I}$ indexed by i.
- The representative household on island i has preferences:

$$E_{i,t}\left[\sum_{s=0}^{\infty}\beta^s\frac{C_{i,t+s}^{1-\gamma}-1}{1-\gamma}\right]$$

where $C_{i,t}$ is the household's consumption in period t, β is their discount factor, and γ is their coefficient of relative risk aversion.

- Expectations are conditional on household i's information set $\Omega_{i,t}$.

Model: Household Constraints

- Households inelastically supply one unit of labor on their island, paid real wage $W_{i,t}$.
- Households own two island-specific assets: the capital $K_{i,t}$ which they rent to firms at $R_{K,i,t}$, and real bonds $B_{i,t}$ which pays risk-free return $R_{B,i,t+1}$.
- Households purchase two goods from an economy-wide market: investment $I_{i,t}$ and consumption $C_{i,t}$. Their budget constraint is

$$W_{i,t} + R_{K,i,t}K_{i,t} + R_{B,i,t}B_{i,t} = C_{i,t} + I_{i,t} + B_{i,t+1}$$

- Investment is used to construct new capital:

$$K_{i,t+1} = I_{i,t} + (1 - \delta)K_{i,t}$$

Model: Firms

There are two types of competitive firms:

1. Intermediate producers produce a different type of good on each island

2. Final goods firms transform intermediates into generic output

Model: Intermediate Goods Firms

- Representative firm on island i produces intermediate $Y_{i,t}$ hiring specialized capital $K_{i,t}$ and labor $L_{i,t}$ at prices $R_{K,i,t}$ and $W_{i,t}$
- Production is Cobb-Douglas with stochastic productivity $A_{i,t}$:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}$$

- Sell output at price $P_{i,t}$

Model: Final Goods Firms

- Representative firm purchases substitutable intermediates $Y_{i,t}$ at prices $P_{i,t}$
- Produce generic output Y_t with CES production function:

$$Y_t = \left(\int_{i \in \mathcal{I}} \phi_{i,t}^{\frac{1}{\eta}} Y_{i,t}^{\frac{\eta-1}{\eta}} \, d\lambda(i) \right)^{\frac{\eta}{\eta-1}}$$

- Demand function, with stochastic shifter $\phi_{i,t}$:

$$P_{i,t} = (\phi_{i,t} \frac{Y_t}{Y_{i,t}})^{\frac{1}{\eta}}$$

- Generic output is used directly for consumption or investment
- **Baseline model**: $\eta \to \infty$ (perfect substitutes)

Model: News

- Islands receive news about their productivity N periods in the future
- Their news signal $S_{i,t}$ is imperfect, due to the "sentiment" shock $\xi_{i,t}$:

$$S_{i,t} = e^{\xi_{i,t}} A_{i,t+N}$$

Model: Shocks

- Productivity and sentiments have aggregate and idiosyncratic components:

$$\ln A_{i,t} = \ln A_t + \ln \hat{A}_{i,t}$$

$$\xi_{i,t} = \xi_t + \hat{\xi}_{i,t}$$

- Independent ARMA processes for all 4 exogenous processes $\ln A_t$, $\ln \hat{A}_{i,t}$, ξ_t, and $\hat{\xi}_{i,t}$

- Idiosyncratic shocks $\hat{A}_{i,t}$ and $\hat{\xi}_{i,t}$ uncorrelated across islands

Model: Information

- Households and firms observe all variables on their island i, as well as an index of average bond rates R_{t+1}:

$$R_{t+1} \equiv \int_{i \in \mathcal{I}} R_{B,i,t+1} d\lambda(i)$$

- Observe 3 signals that inform forecasts of aggregate economy:
  1. Productivity $A_{i,t}$ is a noisy signal of A_t
  2. News $S_{i,t}$ is a noisy signal of A_{t+N} and ξ_t
  3. Index R_{t+1} reveals average expectations
- 3 signals and 4 shocks so the aggregate state is not revealed.
- Information set $\Omega_{i,t}$, with $\omega_{i,t}$ a vector of island-specific prices and quantities:

$$\Omega_{i,t} = \{\Omega_{i,t-1}, A_{i,t}, S_{i,t}, R_{t+1}, \omega_{i,t}\}$$

Model: Equilibrium

Given infinite sequences of exogenous variables $\{\hat{A}_{i,t}, A_t, \hat{A}\xi_{i,t}, \xi_t\}$ for all $i \in \mathcal{I}$, a competitive equilibrium in this economy consists of infinite sequences of prices, $\{W_{i,t}, R_{K,i,t}, R_{B,i,t}\}$ for all $i \in \mathcal{I}$; allocations $\{C_{i,t}, I_{i,t}, K_{i,t}, B_{i,t}, L_{i,t}, Y_{i,t}, Y_t\}$ for all $i \in \mathcal{I}$; and information sets $\Omega_{i,t}$ for all $i \in \mathcal{I}$ such that:

1. Households maximize utility subject to their budget constraints and laws of motion, with expectations formed conditional on their information sets
2. Intermediate firms choose allocations to maximize profits, satisfying their production function and factor demands
3. Final goods firms choose allocations to maximize profits, satisfying their production function and input demands.
4. Investment goods firms make zero profit, satisfying their production function and equilibrium price
5. Exogenous variables follow independent ARMA processes
6. Information sets follow their law of motion
7. The labor market clears: $L_{i,t} = 1$ for all $i \in \mathcal{I}$
8. Risk-free bonds are in zero net supply on each island: $B_{i,t} = 0$

Model: Euler Equations

$$1 = \beta E_{i,t}\left[\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{-\gamma}\left(\alpha A_{i,t+1}K_{i,t+1}^{\alpha-1} + 1 - \delta\right)\right]$$

- When choosing how much to invest, agents forecast their own productivity
- Bonds have the standard Euler equation:

$$1 = R_{i,t+1}\beta E_{i,t}\left[\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{-\gamma}\right]$$

Solution Method

- Linearize the model (lower case variables will denote log deviations)
- Algorithm from Adams (2021): iterative method for solving models with state variables and endogenous signals
  1. Conjecture a process for endogenous information (r_t)
  2. Calculate behavior conditional on information
  3. Find implied process for endogenous information
  4. Repeat until convergence
- Reasonable guesses converge to the unique stable equilibrium

Solution Method: Linearize

- Reduce to two linearized equations with two choice variables: $K_{i,t}$ and $C_{i,t}$
- Other variables islands take as given: $A_{i,t}, R_t, S_{i,t}$
- Lower case variables denote log deviations. Overbars denote steady state values.
- Investment Euler and Law of Motion become:

$$0 = E_{i,t}\left[\gamma(c_{i,t} - c_{i,t+1}) + \beta\overline{R}(a_{i,t+1} + (\alpha - 1)k_{i,t+1})\right]$$

$$k_{i,t+1} = \delta\frac{\overline{Y}}{\overline{I}}(a_{i,t} + \alpha k_{i,t}) - \delta\frac{\overline{C}}{\overline{I}}c_{i,t} + (1-\delta)k_{i,t}$$

- Aggregate bond index:

$$r_t = \int_{i\in\mathcal{I}} E_{i,t}\left[\beta\overline{R}(a_{i,t+1} + (\alpha - 1)k_{i,t+1})\right] d\lambda(i)$$

Solution Method: Stack

- Stack choice variables as $X_{i,t} = (c_{i,t}, k_{i,t})'$, signals as $Z_{i,t} = (a_{i,t}, r_t, s_{i,t})'$
- Use Wold Decomposition to write $Z_{i,t}$ as lag polynomial of its innovations $Z_{i,t} = Z(L)W_{i,t}$
- Write the policy function in terms of innovations: $X_{i,t} = X(L)W_{i,t}$.
- Stack the equilibrium conditions, with B matrices encoding coefficients:

$$0 = E_{i,t}\left[B_{X0}X(L)W_{i,t} + B_{X1}L^{-1}X(L)W_{i,t} + B_{Z0}Z(L)W_{i,t} + B_{Z1}L^{-1}Z(L)W_{i,t}\right]$$

- Use $E_{i,t}[W_{i,t+1}] = 0$ to recursively solve for coefficients of $X(L)$.

Solution Method: Signal Generation

- Signals $Z_{i,t}$ is also a polynomial in fundamental shocks $\epsilon_{i,t}$:

$$Z_{i,t} = S(L)\epsilon_{i,t}$$

- Polynomial $S(L)$ is a sum of exogenous terms $S_Z(L)$ and endogenous terms that depend on the aggregates:

$$S(L)\epsilon_{i,t} = S_Z(L)\epsilon_{i,t} + \int_{i\in\mathcal{I}} [A(L)X(L)]_+ W_{i,t}$$

- In the baseline, $S_Z(L)$ are ARMA processes for $a_{i,t}$ and ξ_t, while $A(L)$ encodes how consumption and expectations determine aggregate asset prices:

$$A(L)\begin{pmatrix} 0 & 0 \\ \gamma(I - L^{-1}) & 0 \\ 0 & 0 \end{pmatrix}$$

Solution Method: Algorithm

1. Conjecture signal polynomial $S^0(L)$
2. For conjectured signal $S^j(L)$, calculate implied policy function $X^j(L)$ using equilibrium conditions
3. Calculate new signal $S^{j+1}(L)$ from signal generation equation
4. If S^{j+1} is not sufficiently close to S^j, return to (2) with new conjecture.

Baseline Calibration

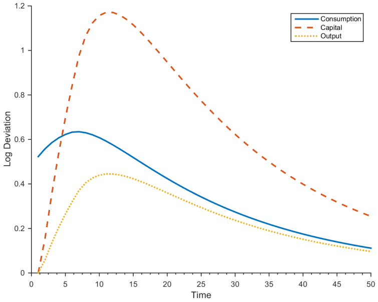
- AR(1) Processes for shocks, estimated from US industry-level productivity estimates

Parameter	Interpretation	Value	Justification
β	Discount factor	0.95	5% Annual real return
δ	Depreciation factor	0.05	NIPA
α	Capital share	0.38	NIPA
γ	Risk aversion	3	Standard value
$\rho_{\hat{a}}$	Persistence of idiosyncratic technology shock	0.80	KLEMS
ρ_a	Persistence of aggregate technology shock	0.78	KLEMS
ρ_ξ	Persistence of idiosyncratic sentiment shock	0.9	Arbitrary
$\rho_{\hat{\xi}}$	Persistence of aggregate sentiment shock	0.9	Arbitrary
$\sigma_{\hat{a}}$	Standard deviation of idiosyncratic technology shock	0.08	KLEMS
σ_a	Standard deviation of aggregate technology shock	0.01	KLEMS
σ_ξ	Standard deviation of idiosyncratic sentiment shock	0.01	Arbitrary
$\sigma_{\hat{\xi}}$	Standard deviation of aggregate sentiment shock	0.01	Arbitrary
N	News delay	5	Arbitrary

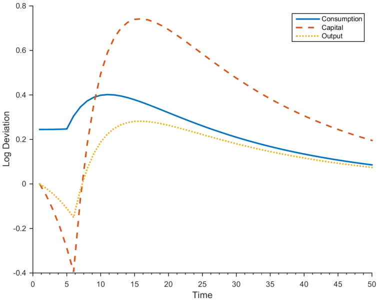
Equilibrium: Island Behavior

- Island exhibits classical responses to classical shocks
  - Productivity increase: raise consumption and investment; output expands
  - News of future productivity increase: reduce investment to smooth consumption; output contracts before productivity is realized
- Unexpected interest rate increase: lower consumption because will likely productivity rise

Equilibrium: Signal Responses

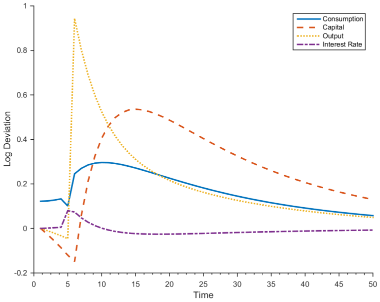


(a) Productivity Signal Innovation

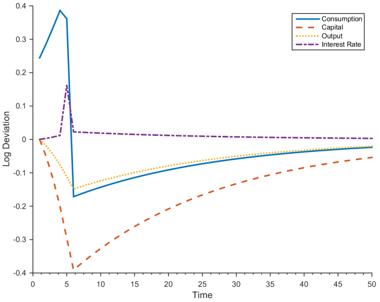


(b) News Signal Innovation

Equilibrium: Aggregate Response



(c) Productivity Shock

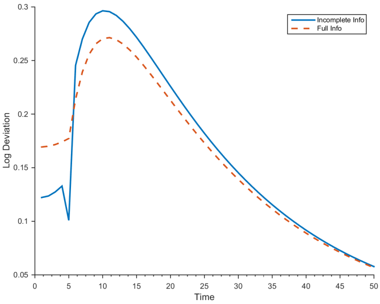


(d) Sentiment Shock

Equilibrium: Inefficiency

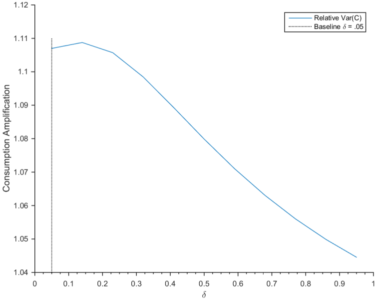
- When islands receive news shocks, they cannot tell how much of the shock is due to future productivity vs. sentiments, or aggregate vs. idiosyncratic components
- Asset prices reveal some information, but may be driven by either sentiments or aggregate productivity
- Islands under-respond to news relative to full information
- Capital propagates: mistakes today affect aggregate output tomorrow

Distortions: Consumption Response



- Response to aggregate productivity shock is distorted
- *Constrained full information*: observe aggregates (versus omniscient observation of ξ_t)
- Aggregate amplification relative to full information: 110.7% consumption volatility

Distortions: Capital Amplification



- Inclusion of capital increases amplification
- Mistakes "pile up"
- Under-investing from a bad sentiment shock lowers income persistently
- Other parameters raise amplification:
  - Risk aversion
  - Idio. sentiment variance
  - Idio. sentiment persistence
  - News delay

Government Policy

- The government can intervene in the bond market to improve over the inefficient equilibrium
- Not obvious how government affects asset prices
  - Example: 2020 Fed SMCFF program buys corporate bonds and ETFs
  - Large price changes from announcement effect; actual purchases are small (Gilchrist et al 2020)
- Agnostic about how the government actually moves bond prices; introduce an intertemporal wedge
- Consider a perpetual policy rule (vs. Ramsey plan)

Investment Wedge Implementation

- Introduce wedge τ_t into Euler equations:

$$1 = (1 + \tau_t) R_{i,t+1} \beta E_{i,t} \left[\left(\frac{C_{i,t}}{C_{i,t+1}} \right)^{-\gamma} \right]$$

$$1 = (1 + \tau_t) \beta E_{i,t} \left[\left(\frac{C_{i,t}}{C_{i,t+1}} \right)^{-\gamma} \left(\alpha A_{i,t+1} K_{i,t+1}^{\alpha-1} + 1 - \delta \right) \right]$$

- Consider a simple rule: wedge τ_t linear function of economic aggregates
- One approach: restrict to publicly available information
  - If government knows true state of the world, optimal response is to reveal all information
  - If government knows more than islands but not everything, could be interesting tradeoffs between partial revelation and creating distortions

Optimal Policy Rule using Bond Prices

- Restrict policy to be linear in bond prices:

$$\tau_t = b_r r_{t+1}$$

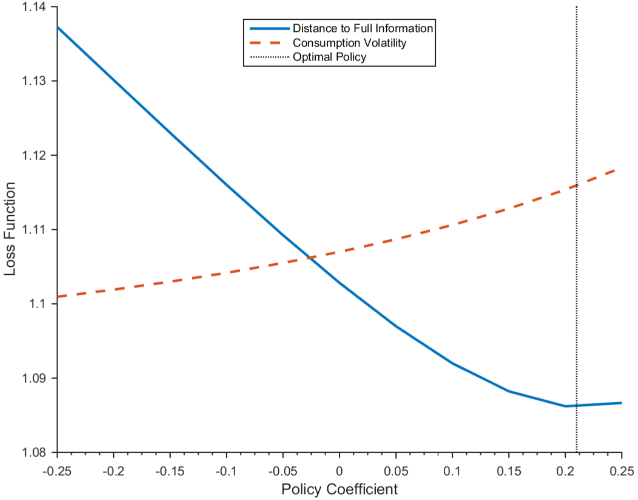
- What is the optimal parameter b_r governing taxes/subsidies? Need an objective function

- True welfare maximization impossible in a linear model

- Compromise: Minimize distance of aggregate consumption IRF $\vec{c}(b_r)$ to *omniscient full info* IRF $\hat{\vec{c}}$:

$$||\vec{c}(b_r) - \hat{\vec{c}}||_2$$

Policy Rule using Bond Prices: Optimization



Optimal Policy Rule using Aggregate Quantities

- Restrict policy to be linear in aggregate output or investment:

$$\tau_t = b_y y_t \quad \tau_t = b_i i_t$$

- This introduces new information; islands could not observe aggregates before
- How to model?
  - Do not reveal the wedge τ_t: no additional signals, no revelation (*today's results*)
  - Let them see the wedge τ_t, add another shock to prevent revelation (e.g. $\eta < \infty$)

Optimal Policy Parameters

Policy Input	Coefficient	Value	Distance Change	Cons. Volatility Change
Bond Prices	b_r	0.21	-3%	1%
Output	b_y	0.08	-1%	≈0%
Investment	b_i	-0.29	-5%	-2%

Policy Rule Discussion

- Bond price rule:
  - $b_r > 0$ implies policymakers should move *with* asset prices
  - Elasticity is modest: increase volatility of effective returns by 21%
- Output rule:
  - $b_y > 0$ implies policymakers should make asset prices more procylical
  - Elasticity is modest: reduce effective returns by 0.08 p.p. when output rises by 1%
- Investment rule:
  - $b_i < 0$ implies policymakers should lower effective returns when investment is high
  - Elasticity is large: raise effective returns by 0.29 p.p. when investment rises by 1%
- Intuition:
  - After news shock, expected returns are high
  - ... but not as high as under full information
  - Optimal policy: encourage investment when asset prices are low

Conclusions & Next Steps

- Incomplete information amplifies effects of capital cost shocks
- Optimal policy encourages investment when asset prices are low, discourages when asset prices are high
- Can generalize model, but these ingredients do not clearly change conclusions
  - Elastic labor supply
  - Investment adjustment costs
  - Islands produce imperfect substitutes
- To do: more interesting policy rules, microfounding asset purchases?