Terms of Trade Shocks and Heterogeneous International Portfolio Positions*

Jonathan J Adams†**and Philip Barrett**‡

PRELIMINARY DRAFT This version: November 30, 2018

Link to Most Current Version

Abstract

How do terms of trade shocks affect open economies? We use a panel of global commodity prices to estimate the dynamic effects of terms of trade shocks on macroeconomic variables for 205 countries. We find that terms of trade shocks resemble wealth shocks: a terms of trade improvement increases consumption and investment by more than output and decreases net exports, contrary to prior evidence and standard theory. To explain this outcome, we also show that terms of trade improvements increase countries' net foreign asset position, due to valuation effects of nominal net assets. To make sense of these results, we augment a standard business cycle model with realistic international portfolio choice. We estimate the model for a large sample of countries, and show that it can replicate our empirical findings: terms of trade improvements look like wealth shocks, and their importance for business cycles is heterogeneous, depending on the country's international portfolio position.

JEL-Codes: F30, F41

Keywords: Terms of trade, country portfolios, international business cycles, small open economies, home bias, gross asset positions, exchange rates

*We are grateful for helpful comments from our colleagues, and seminar participants at the EES/EEA meetings, the 2nd CEBRA Workshop on Commodities and Macroeconomics, and the University of Florida economics seminar. The views expressed in this paper are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

†University of Florida. Website: www.jonathanjadams.com Email: adamsjonathan@ufl.edu

‡International Monetary Fund. Website: www.pobarrett.com Email: pbarrett@imf.org

1 Introduction

How do terms of trade shocks affect macroeconomies? In traditional business cycle models (e.g. Mendoza 1995) terms of trade improvements act like productivity improvements, where income effects lead households to consume and invest more, but with substitution effects such that households work more and output increases by more than consumption. We challenge this conventional wisdom. Instead, we argue that terms of trade shocks resemble wealth shocks more than productivity shocks, and demonstrate a channel through which this occurs: nominal exchange rate exposure in countries' international asset positions.

First, we examine the effects of country-specific terms of trade shocks on a variety of macroeconomic aggregates, and find results that resemble wealth effects. The shocks we consider are unexpected changes to a panel that we construct of country-specific commodity terms of trade indices. The traditional empirical methodology is to consider shocks to the measured terms of trade, but this may include may include endogenous factors on both the import and export side. Instead, world commodity prices are mostly homogenous and more likely to be exogenous to countries with small market shares, as argued by Chen and Rogoff (2003). We run a series of dynamic panel regressions and find that terms of trade shocks look like a wealth shock, increasing consumption, investment, and imports by more than output[1]. This strategy is related to the approaches of Fernandez, Gonzalez, and Rodriguez (2018), Bet and Peluffo (2017), and Shousha (2015), who construct country-specific indices of commodity export prices and analyze their effects on business cycles. Our indices improve upon these approaches in three ways: our panel is longer, stretching from 1960 to 2017; the panel is wider, covering 203 countries; and the index captures the entire terms of trade, instead of only export prices.

Second, we document a source for large wealth effects from terms of trade shocks: nominal exchange rate exposure in countries' international asset position. Most countries choose portfolios of nominal assets and liabilities that are long in their domestic currency and short in foreign currency. In the Benetrix, Lane, and Shambaugh (2015) panel of international asset positions, 107

[1]Other papers have also discovered this volatile consumption response, including Drechsel and Tenreyro (2017) in Argentinian data, and Fernandez, Gonzalez, and Rodriguez (2018) in an estimated small open economy model,

of 116 countries have this type of portfolio position, such that their net asset positions gains value when their exchange rate appreciates[2]. When we examine the effects of terms of trade shocks on international portfolio positions, we find that a terms of trade improvement appreciates a country's currency, increasing that country's net asset position because of the portfolio's currency exposure. Corroborating this channel, we show that the effect occurs in economies with flexible exchange rates; fixed exchange rate economies instead exhibit traditional responses to terms of trade shocks.

Third, we consider a small open economy model with many potential sources of aggregate fluctuations. We augment this standard model with a meaningful portfolio decision and show that countries with larger exchange rate exposure in their international portfolio will exhibit higher business cycle volatility. In particular, we allow for trend shocks to productivity as in Aguiar and Gopinath (2007), who argue that these shocks are important source of variation for developing countries. This is an important inclusion because we demonstrate that terms of trade shocks can cause consumption to be more volatile than output, which could also be explained by trend shocks. We estimate the model for many small open economies, and show that in countries where portfolios are more exposed to exchange rates, consumption is more sensitive to terms of trade shocks. Given the variety of exchange rate exposures observed in international portfolio positions, our findings suggest that we should consider the importance of terms of trade shocks for business cycles to be heterogeneous across countries.

This paper joins a large literature assessing the importance of terms of trade shocks for macroeconomic fluctuations. There is no general consensus. Traditional business cycle models such as Mendoza (1995) and Kose (2002) find that terms of trade shocks represent a large source of fluctuations. But less structural analysis, often using SVARs as in Broda (2004) and Schmitt-Grohe and Uribe (2017), estimate the contribution of terms of trade shocks is small[3]. Our model

[2]This panel covers 1990-2012, and Adams and Barrett (2017) report that the average exchange rate exposure is 32% of GDP, such that a 1% appreciation in a country's currency would increase its net asset position by 0.32%, *ceteris paribus*.

[3]Although this generalization is not perfect: Fernandez, Schmitt-Groh, and Uribe (2017) show that a SVAR with disaggregated world prices will assign more importance to terms of trade shocks, while Zeev, Pappa, and Vicondoa (2017) argue that accounting for news shocks to terms of trade increases their estimated effect.

estimation uses Bayesian methods like the SVAR literature, we find that the contribution of terms of trade shocks to macroeconomic volatility is small. However, our analysis differs in that we estimate the model separately for many countries, and find that the importance of terms of trade shocks is heterogeneous and depends on the country's international portfolio position.

The remainder of the paper is organized as follows. Section 2 estimates macroeconomic responses to the commodity terms of trade shocks. Section 3 presents the small open economy model with gross asset positions. Section 4 presents the model results and estimates for many countries. Section 5 characterizes the countries' portfolio selection problem and compares the solution to the data. Section 7 concludes.

2 Empirical Analysis

How do terms of trade shocks affect economies? In this section, we address this question by estimating the dynamic effects of plausibly exogenous terms of trade shocks on various macroeconomic variables.

2.1 The Commodity Terms of Trade Index

Exogenous shocks to a country's terms of trade are difficult to isolate, because the price of many traded goods may be endogenous. To resolve this problem, we create indices of the terms of trade for commodity goods, for which the prices could be considered exogenous for a small open economy. Commodities are plausibly exogenous because they are relatively standardized around the world, unlike traded services such as tourism or manufactured goods that are differentiated across countries. In some cases the assumption that commodity prices are exogenous may not be valid, so we will have to omit some countries from our analysis.

The data for our commodity terms of trade indices come from two sources. Prices are from the World Bank's CMO Commodity Price Data (World Bank, 2018), which include monthly prices or indices for 74 commodities, with coverage for some since 1960. The CMO also calculates indices for six aggregated commodity groups: food, beverages, fuels, raw materials, metals, and gold. Trade data are from the UN's COMTRADE database.

The commodity terms of trade index is the ratio of a commodity export price index to a commodity import price index, that is:

$$TOT_{it}^{commoditiy} = \frac{\sum_c X_{i,t}^c P_{ct}}{\sum_c M_{i,t}^c P_{ct}}$$

where P_{ct} is the world price of commodity c at time t, normalized by its geometric mean over the sample period. $X_{i,t}^c$ and $M_{i,t}^c$ are the average export and import weights of commodity c in country i over the sample period. In our baseline index, the weights are the average shares of commodity c in country i's exports or imports over the previous 5 years. We construct our commodity terms of trade index using 41 commodity price indices from the CMO data and total trade in the same categories from the COMTRADE data. The categories covered by this index are important, making up essentially all global trade in commodities[4].

We also drop the USA and Saudi Arabia from our sample, as the exogeneity assumption may be weak here. More concretely, these are the only two countries with at least 10% export of their exports in a commodity where they make up at least 10% of global trade. That is, a good whose prices on world markets might be affected by domestic conditions, and that is a major component of trade. This criterion is met by energy commodities in the case of Saudi Arabia, and for several products in the case of the USA. Countries where exogeneity in some goods might be violated are retained, but only if those goods are small shares of trade. For example, although France accounts for more than 10% of global trade in beverages, these products count for only around 2% of French exports.

To isolate the shocks to this index, we estimate autoregressive processes for $TOT_{it}^{commodity}$ for each i, with lag length determined by the Akiake Information Criterion, denoting the residuals Z_{it}. The estimated residuals are then our terms of trade shock series, Z_{it}. Further details of the shock estimation can be found in Appendix B.1.

[4]Although not all global trade. According to UN COMTRADE data, manufactured goods constituted some 76% of goods trade in 2016.

2.2 Statistical model

We consider eleven key macroeconomic variables - the "full" terms of trade (as distinct from the commodity terms of trade), output, consumption, investment, exports, imports, aggregate hours, net exports, nominal and real effective exchange rates, and net foreign assets. In most cases, the data are from national accounts, as collected and standardized by Feenstra, Inklaar, and Timmer (2015). Net foreign assets data are computed relative to GDP, and come from Benetrix, Lane, and Shambaugh (2015). Our dataset covers 203 countries, with annual observations covering 1972-2014 for most variables.[5] Appendix A.1 lists the countries and the time coverage for each variable.

We chose these data series because standard models of the propagation of terms of trade shocks have specific implications for these variables. In canonical international macroeconomic models, terms of trade shocks are transmitted through trade linkages. As a result, terms of trade shocks are very similar to productivity shocks. This occurs because exports are entirely produced domestically, but consumption and investment are composed partly of imports. So when the terms of trade improves, domestic firms are more productive, in the sense that the output of given capital and labor can be exchanged for more real investment and consumption goods. And so the dynamic responses of these models feature productivity-shock-like properties. In particular, output and hours increase by more than consumption, at least in the short term, as the substitution effect from higher real wages outweighs the income effect. The increased return on capital stimulates an investment boom. And because output increases by more than consumption, and because consumption is partly imports, exports increase by more than imports. By testing whether these predictions hold true in the data, we hope to assess the standard view of how terms of trade shocks propagate.

To evaluate the dynamic effects of a terms of trade shock on a macroeconomic quantity X_t, we regress the log difference of X_t on lags of itself, as well as several lags of the log difference of shock. We estimate each equation separately, deviating from the typical approach of estimating

[5] Data are available going back to 1960, but the end of the Bretton Woods era in 1972 results in clear series breaks for many data series, particularly commodity prices.

a panel VAR for two reasons. First, with a plausibly exogenous shock, we can consistently estimate the impulse response function without needing to make identifying assumptions about the structure of the matrices in a VAR. Secondly, our estimation is more parsimonious than a VAR, which estimates many cross-terms in the coefficient matrices, allowing spurious estimates to affect calculated dynamic effects. Nevertheless, we estimate a panel VAR in Appendix E and find similar results.

2.2.1 Dynamic Model

Our primary method for identifying the effects of a terms of trade shock is to estimate an ARMA model. For a macroeconomic quantity $X_{i,t}$ in country i, our headline specification is a regression of the following form:

$$X_{i,t} = \mu_i + \eta_t + \sum_{j=1}^{N} \beta_j X_{i,t-j} + \sum_{j=0}^{M} \gamma_j Z_{i,t} + \varepsilon_{t,i} \tag{1}$$

where $Z_{i,t}$ shock to the log of the commodity terms of trade index, ε_t is an error term, μ_i are country-specific constants, and η_t is a common time fixed effect. So X_t is modeled as an ARMA(N, M) process in the terms of trade shock after accounting for country and time fixed effects. The model is estimated via OLS.

2.2.2 Impulse Responses and Total Dynamic Effect

The dynamic response impulse response of a quantity X_t to a shock Z_t is calculated by equation 2. The predicted response in period $t + k$ is given by:

$$\Delta \hat{X}_{t+k} = \alpha + \gamma_k \Delta Z_t + \sum_{j=1}^{\min(N,k)} \beta_j \Delta \hat{X}_{t+k-j} \tag{2}$$

The estimated impulse response to a one percent increase in the terms of trade index are reported in Figure 1a. Standard errors on the impulse responses are computed from applying the delta method to the standard errors on the regression coefficients[6]. The first panel to note, labeled ToT, is the response of the terms of trade to the commodity-based terms of trade index. This responds by almost around 0.2 percentage points on impact and reflects the fact that, for most

[6]Explicit formulae for these are provided in Appendix B.2

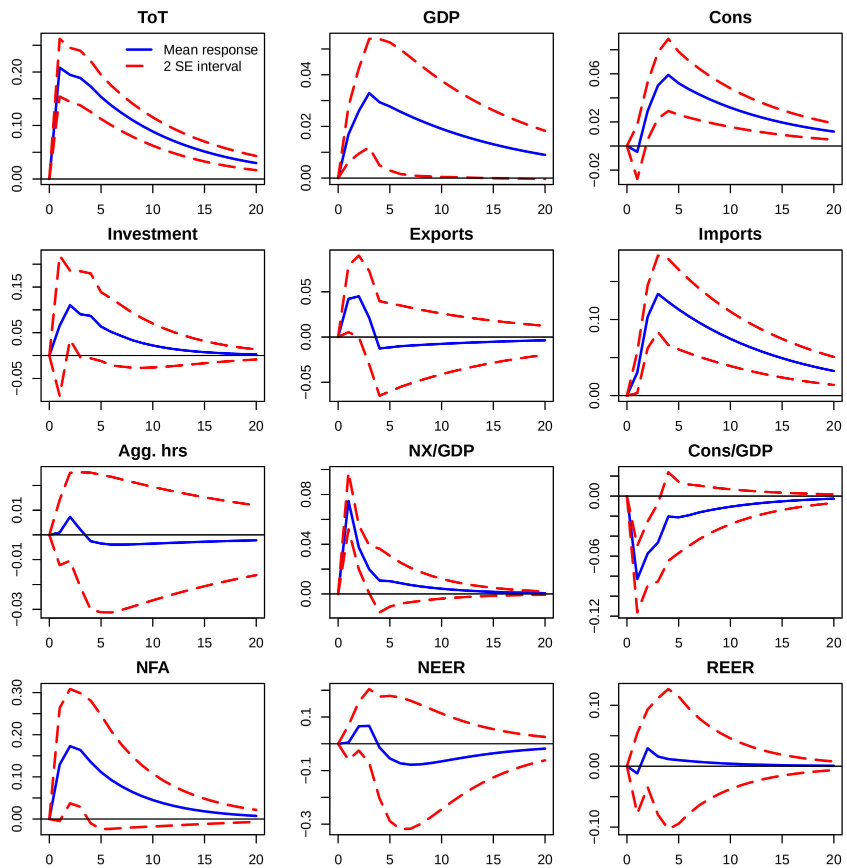
countries, the commodity terms of trade has a less than 100% pass-through to the terms of trade itself. The slow decay then reflects the fact that the shock is the estimated innovation from fitting an autoregressive process to the index.

The remaining panels in Figure 1a pose three challenges to the canonical view of terms of trade shocks. First, that the increase in output is barely statistically significant, even at its peak. In contrast, that the shock elasticity of consumption is about twice as large (0.24 vs. 0.12 for GDP), and clearly statistically significant. As a measure of the relative effect, we estimate the response of the real consumption-output ratio. This is also positive and, at its peak, (just) statistically significant. Second, that the response of labor input, measured by aggregate hours worked, is also statistically indistinguishable from zero at all horizons. Moreover, neither of the two components of aggregate hours - employment nor average hours (not shown) - feature a statistically significant response. As a means of comparing this to the consumption response, we also plot the impulse response for the ratio of consumption to total hours worked. This is positive throughout and statistically significant near its peak. Third, that the response of imports to a terms of trade shock is large and positive whereas the response of exports is statistically insignificant at (almost) all horizons. All three of these observations represent challenges to the canonical view of the propagation of terms of trade shocks.

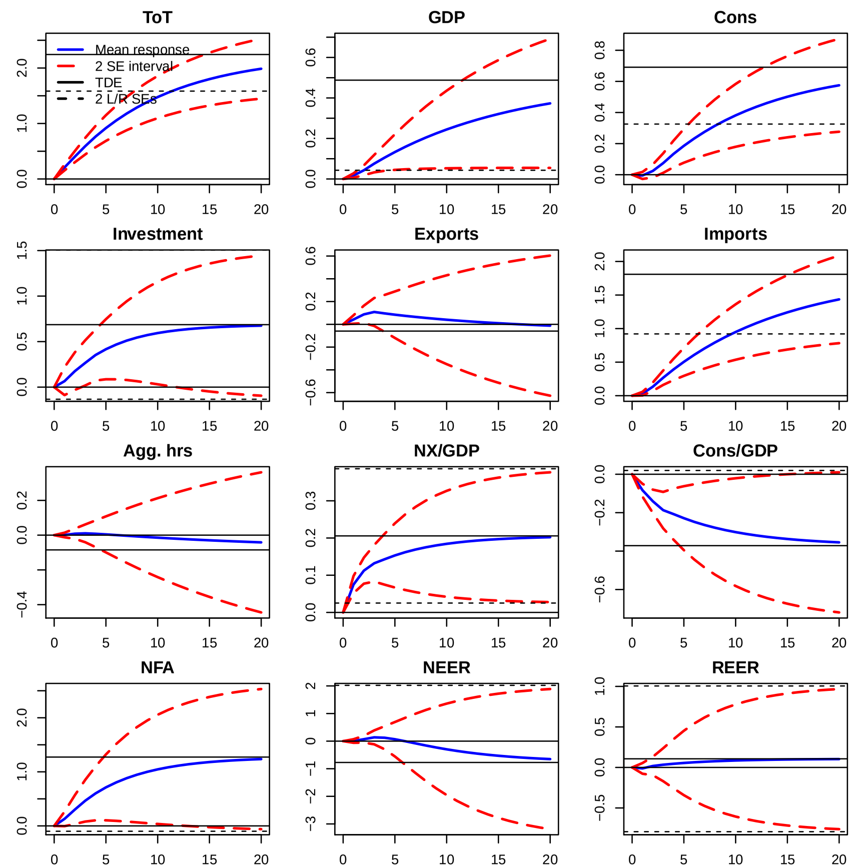
We summarize the macroeconomic responses by estimating the *Total Dynamic Effect* (TDE). This is the long-run cumulative impacts of the terms of trade shock, and for the quantity X_t is given by

$$\begin{aligned} TDE_X &= \sum_{k=0}^{\infty} \Delta \hat{X}_{t+k} \\ &= \frac{\sum_{j=0}^{M} \gamma_j}{1 - \sum_{j=1}^{N} \beta_j} \end{aligned} \tag{3}$$

In this example, the TDE is the cumulative effect of a temporary shock, and is the limit of the cumulative responses shown in Figure , but should also be familiar from frameworks where the shock is permanent and the model is estimated in growth rates. Standard errors of the TDE are also computed using the delta method, details of which can be found in Appendix B.2.



(a) Instantaneous



(b) Cumulative

Figure 1: Estimated Impulse Responses to a one percent commodity Terms of Trade shock

TDEs are also included in Table 1. These confirm that the cumulative consumption response is positive and highly statistically significant in the full sample, and much larger than the output response. The responses for real imports and, to a lesser extent, investment are positive and highly statistically significant. Those for exports and aggregate hours are not.

One of the advantages of summarizing the responses as TDEs is that we can easily compare results from different subsets of the data. To that end, Table 1 also include results when the sample is restricted to times and countries where the the exchange rate is flexible and when it is pegged (we use the Klein and Shambaugh (2010) definition of currency pegs). While the decision to switch between exchange rate regimes may be endogenous to the properties of the terms of trade shock, comparing the results across these two subsamples is at least indicative of the mechanism driving the response to a terms of trade shock. These results show that the income-effect-like response is more pronounced when exchange rates are flexible than when pegged, with larger relative output and investment responses when exchange rates are not free to move. This is consistent with earlier findings (Broda, 2004) that terms of trade shocks have larger effects on real GDP in countries with fixed exchange rates.

The results are also largely robust to cutting them by country income level. In all cases, the point estimate for the consumption response is larger than for output, imports increase, and aggregate hours and net exports are both either insignificant or decrease.

Table 1: Total dynamic effect, OLS estimation: All, 1972-2017 (203 countries)
Arithmetic disaggregated, year and country FEs, Macroeconomic variables

Sample	ToT	GDP	Cons	Investment	Exports	Imports	Agg. hrs	NX/GDP	NA	NFA	NEER	REER
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
All	2.244***	0.488**	0.691***	0.686*	-0.059	1.809***	-0.085	0.206**	-0.372*	1.273*	-0.773	0.106
(N=203)	(0.33)	(0.222)	(0.183)	(0.41)	(0.412)	(0.444)	(0.342)	(0.09)	(0.196)	(0.686)	(1.397)	(0.45)
NFA bottom 25%	1.685***	0.858**	0.839***	1.821**	-0.479	1.512	-0.27	-0.152	-0.32	4.297*	2.064	0.026
(N=27)	(0.647)	(0.354)	(0.235)	(0.831)	(0.529)	(1.015)	(0.388)	(0.346)	(0.312)	(2.356)	(2.191)	(0.292)
NFA top 25%	2.256***	0.353	1.145	0.336	0.61	3.517**	0.027	0.376**	-0.655	-2.739	0.747	0.93
(N=27)	(0.496)	(1.008)	(1.056)	(0.862)	(1.787)	(1.668)	(0.199)	(0.164)	(0.494)	(3.425)	(1.636)	(1.213)
NFDA bottom 25%	1.574**	1.178***	0.93***	1.958**	0.184	1.745*	0.238	0.161	-0.452**	4.721*	2.222	-0.031
(N=28)	(0.628)	(0.359)	(0.272)	(0.85)	(0.72)	(1.054)	(0.828)	(0.106)	(0.209)	(2.61)	(1.928)	(0.268)
NFDA top 25%	1.852***	-0.048	1.948	-0.044	0.347	4.317**	-0.54*	-0.341	-0.03	-1.115	0.205	1.785
(N=27)	(0.553)	(1.133)	(1.565)	(0.519)	(2.224)	(1.936)	(0.299)	(0.621)	(0.564)	(0.785)	(3.298)	(2.057)
NFEA bottom 25%	1.571**	1.088	1.101***	0.4	0.03	2.153**	-0.069	-0.047	-0.466	1.692	-0.448	0.198
(N=27)	(0.645)	(0.801)	(0.419)	(0.666)	(1.255)	(1.062)	(0.71)	(0.441)	(0.402)	(1.837)	(0.997)	(0.423)
NFEA top 25%	1.635***	1.298	0.576	1.894***	0.546	1.403	-0.356	0.395***	-0.734**	-3.512	1.972	-0.405
(N=27)	(0.609)	(0.836)	(0.438)	(0.709)	(1.158)	(1.272)	(0.506)	(0.105)	(0.329)	(3.659)	(1.981)	(0.285)
NFCDA bottom 25%	1.577**	1.143***	0.899***	1.878**	0.2	1.701*	-0.348	0.163	-0.444**	4.424*	1.635	0.002
(N=28)	(0.622)	(0.353)	(0.265)	(0.83)	(0.687)	(1.026)	(0.445)	(0.103)	(0.204)	(2.481)	(2.084)	(0.29)
NFCDA top 25%	2.398***	0.239	1.349	-0.11	1.19	3.453**	-0.49	0.067	-0.599	-0.767	-0.463	0.73
(N=28)	(0.843)	(0.958)	(0.85)	(0.682)	(2.059)	(1.532)	(0.383)	(0.598)	(0.556)	(0.856)	(1.867)	(1.425)

Note: $p < 0.1^{*}$; $p < 0.05^{**}$; $p < 0.01^{***}$
Robust standard errors reported in parentheses

These results stand in contrast to much of the literature, which typically find that output rises and consumption falls in response to a terms of trade improvement (Schmitt-Grohe and Uribe, 2017). We also find evidence that supports the standard Harberger-Laursen-Metzler effect (Harberger (1950) and Laursen and Metzler (1950)), in which terms of trade improvements increase net exports.

If the standard model of the propagation of terms of trade shocks is unable to explain these results, what might? Our interpretation is that terms of trade shocks resemble wealth shocks, as consumption, investment, and imports all increase without corresponding increases in output, hours, and exports. But if countries respond to a terms of trade improvement by increasing output less than consumption and by increasing exports less than imports, then how does their wealth increase? We argue that trade shocks have an important valuation effect on the country's net asset position. In particular, nominal exchange rates fall (so their currency appreciate), improving the net foreign asset position if the country has home bias in nominal bonds, as most countries do (Adams and Barrett, 2017).

We find that a terms of trade improvement causes a country's exchange rate to fall, or equivalently, its currency to appreciate. This is a well established relationship, particularly for real exchange rates (see for example Amano and van Norden (1995), Broda (2001), or Schmitt-Groh and Uribe (2018)). The nominal rate is what matters for our exchange rate valuation channel, but given their close correlation (Rogoff, 1996), it is unsurprising that it moves in the same direction as the real exchange rate after a terms of trade shock. We use monthly data to estimate the size of this effect in section 2.4.

There is further evidence that the exchange rate valuation channel is the cause of the wealth effect. When we consider the effects of terms of trade shocks on countries with fixed exchange rates, the puzzle disappears and the response resembles a TFP shock: consumption increases by less than output, net exports rise, and individuals work more hours. Rather, it is flexible exchange rate economies that drive the documented puzzle; when currencies can appreciate after a terms of trade improvement, then consumption rises more than output and the dynamic effects resemble a wealth shock.

2.3 Local Projections

Our second tool for evaluating the dynamic effects of a terms of trade shock on a macroeconomic quantity X_t is a local projection, following Jorda (2005). For each variable, we estimate separately the following regression for each horizon $h = 0, 1, \ldots, H$:

$$X_{i,t+h} = \mu_i^h + \eta_t^h + \beta^h Z_{i,t} + \sum_{j=1}^{N} \rho_j^h X_{i,t-j} + \varepsilon_{t,i} \tag{4}$$

where $Z_{i,t}$ shock to the log of the commodity terms of trade index, ε_t is an error term μ_i are country-specific constants, and η_t is a common time fixed effect.[7] The coefficients β^h trace out an impulse response for the variable X_t. For simplicity, our headline specification only conditions on past lags of the dependent variable. In Appendix XXYYZZ we expand the set of controls, showing that the basic results are the same.

While the local projection specification has many advantages – such as robustness to mis-specification that dynamic models may suffer from – it is relatively data-intensive. To estimate an h-period ahead response, one must have at least $h+M$ periods of data available, covering both the lagged right-hand-side variables and the the h-period ahead outcome. In order to estimate at least ten years of impulse responses, we therefore restrict our sample to countries with at least 20 annual observations. This requirement restricts our sample to a balanced panel of 47 countries, each with 46 observations.

2.3.1 Projection Results

Figure 2 presents the results of estimating the local projection specification decribed in equation 4. Heteroskedasticity-consistent robust confidence intervals are reported in parentheses. The left-hand panel presents the period-by-period impulse responses and the right hand panel the cumulative ones (which are computed by replacing the dependent variable in equation 4 with the sum between t and $t + h$). The units are log points for all variables except NFA, which is

[7] Note that estimating country fixed effects separately for each h will provide results that are robust to country-specific linear time trends.

expressed in percent of GDP, and hence can be interpreted as the elasticity of the response to a one percent improvement in the commodity terms-of-trade.

The first response to note, labeled ToT, is the response of the terms of trade to the commodity-based terms of trade index. This increases by almost around 0.5 percentage points on impact and reflects the fact that, for most countries, the commodity terms of trade has a less than 100% pass-through to the terms of trade itself. The effect is very persistent, with a half-life of approximately ten years.[8]

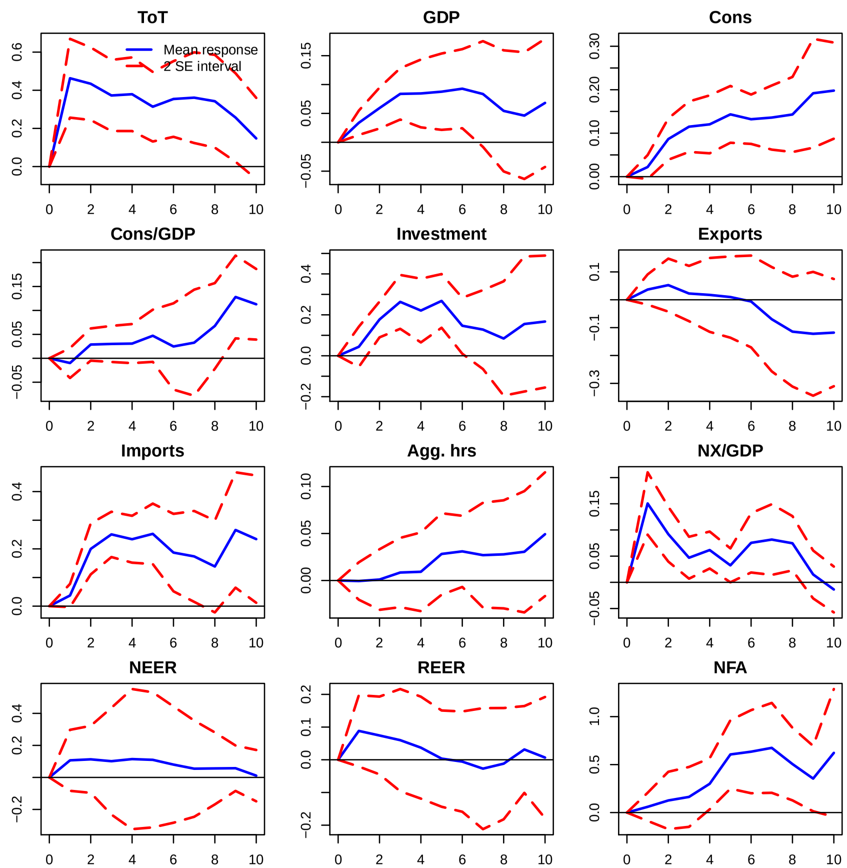
The remaining panels in Figure 2a pose three challenges to the canonical view of terms of trade shocks. First, that the increase in output is only convincingly statistically significant at short horizons, and the cumulative effect only slightly different from zero. This is at odds with the view of terms of trade shocks acting principally through trade and investment, as such a persistent improvement should translate to a sustained boom in output. In contrast, the consumption increase is highly significant at all horizons. Moreover, it is about 50 percent larger, with a cumulative increase of around 1.5 percent over ten years (versus 1 percent for GDP).

Second, that the response of imports is large, positive, and highly persistent, whereas movements in exports are statistically indistinguishable from zero. The net export ratio – nominal exports minus nominal net imports divided by nominal GDP – increases. This is due to the price effect, which increases the relative value of exports even though their volume is unchanged.

Third, the response of labor input, measured by aggregate hours worked, is also statistically indistinguishable from zero at horizons less than five years (and thereafter only barely so). Furthermore, neither of the two components of aggregate hours - employment nor average hours (not shown) - feature a statistically significant response.

The impulse responses above show some weak evidence consistent with our story – nominal and real exchange rates rise on impact, and net foreign assets also increase. Yet the effects are not statistically significant, so in the following sections we provide further evidence for our view, documenting that 1) monthly nominal exchange rates appreciate following terms-of-trade

[8] The likelihood that there may be a unit root in the data is another reason to use local projections, which are robust to this.



(a) Instantaneous

(b) Cumulative

Figure 2: Estimated Impulse Responses to a one percent commodity terms of trade shock

shocks, 2) countries with larger net foreign asset positions experience larger income effects, and 3) countries with flexible exchange rates also exhibit larger income effects.

2.4 Evidence on nominal exchange rates

That a terms of trade improvement causes a country's currency to appreciate is a well established relationship, particularly for real exchange rates (see for example Amano and van Norden (1995), Broda (2001), or Schmitt-Groh and Uribe (2018)). The nominal rate is what matters for our exchange rate valuation channel, but given their close correlation (Rogoff, 1996), one would expect that the nominal and real exchange rate will move in the same direction following a terms of trade shock.

The impulse responses estimated above do feature nominal and real exchange rate appreciation, yet the effect is not statistically significant. As exchange rates are volatile and quick to response to news, it is unsurprising that annual data fail to sharply identify the anticipated effect. To identify the effect of our terms of trade shocks on exchange rates, therfore we turn to higher frequency data.

We construct a monthly series for the commodity terms of trade indices using the monthly prices from the World Bank's CMO Commodity Price Data (World Bank, 2018). We take monthly nominal effective exchange rates from the IMF's IFS database (IMF, 2018) and from the World Bank, which are a series of trade-weighted exchange rates. The two series have coverage of different countries and years, so we use both series and take the geometric mean when both are available. This raises the number of exchange rate observations from 33,787 (IMF) or 37,529 (World Bank) to 43,936, although using either subset does not substantially change our estimates. Unlike our annual dollar exchange rates, the nominal effective exchange rate is measured in units of the basket of foreign currencies relative to the domestic currency. We also take interest rates on bank deposits from the IFS database. Monthly measurements of terms of trade on all traded goods are taken from the World Bank.

To estimate the effect of terms of trade shocks, we regress monthly exchange rate growth on monthly terms of trade growth, using the following specification:

$$\Delta\mathcal{E}_{i,t} = \Delta Z_{i,t} + IR_{i,t-1} + \mu_i + \kappa t + \varepsilon_{i,t} \tag{5}$$

where $\mathcal{E}_{i,t}$ denotes country i's nominal effective exchange rate in period t, $Z_{i,t}$ is the commodity terms of trade index, μ_i is a country fixed effect, and κ is the monthly trend. The Δ operator denotes the monthly log difference, so that e.g. $\Delta\mathcal{E}_{i,t} \equiv \log\mathcal{E}_{i,t} - \log\mathcal{E}_{i,t-1}$. To measure the valuation effect, we are interested in the instantaneous response of exchange rates, versus the long run response reported in Table **??**, given that after a shock, the expected future response would be reflected in current asset prices. The interest rate control is important, as high domestic interest rates compensate for expected currency devaluation.

Table 2 reports our results. The OLS estimates in column (1) are statistically indistinguishable from zero, reflecting the endogenity issues when using measured terms of trade. Column (2) reports the response to a commodity terms of trade improvement, which causes a small appreciation. Because this exchange rate is measured in units of the basket of foreign currencies relative to the domestic currency, a positive coefficient indicates that a terms of trade improvement leads to a currency appreciation. When controlling for the domestic as in column (3), the response is much larger, and the sign on the interest is as expected: higher interest rtes predict currency devaluation. Columns (4) and (5) respectively add controls for country fixed effects and a linear time trend. Lastly in order to understand the size of the effect, in column (6) we instrument for measured terms of trade with commodity terms of trade, so that the estimated coefficient can be interpreted as the response to measured terms of trade instead of commodity terms of trade, which is much more volatile. This regression limits our sample size because the monthly measured terms of trade have limited coverage, so standard errors are large for the two-stage regression, but this gives a back-of-the-envelope figure for the magnitude of a general terms of trade shock. Accordingly, a 10% terms of trade improvement leads to a 2.3% currency appreciation. The size of the implied valuation effect is large. The average country gains 0.32% of GDP in valuation on nominal assets after a 1% exchange rate appreciation (Adams and Barrett, 2017), so a 10% terms of trade improvement leads to a valuation effect worth 0.73% of GDP.

Table 2: Impact of terms-of-trade shocks on nominal effective exchange rates: monthly data

	(1) Meas. ToT	(2) Comm. ToT	(3) Int. Rate Control	(4) Country F.E.	(5) F.E. + Trend	(6) IV: Meas. ToT
Δ Measured Terms of Trade	4.6e-04					.227
	(0.896)					(0.151)
Δ Commodity Terms of Trade		.0079	.0155**	.0142*	.0139*	
		(0.314)	(0.039)	(0.052)	(0.057)	
Lagged Interest Rate			-1.5e-05***	-1.3e-05***	-1.3e-05***	
			(0.000)	(0.000)	(0.000)	
Constant	-.002***	-.0023***	-.0017***	-.0017***	-.0104***	-.0021***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	11029	30032	19203	19203	19203	11029

p-values in parentheses

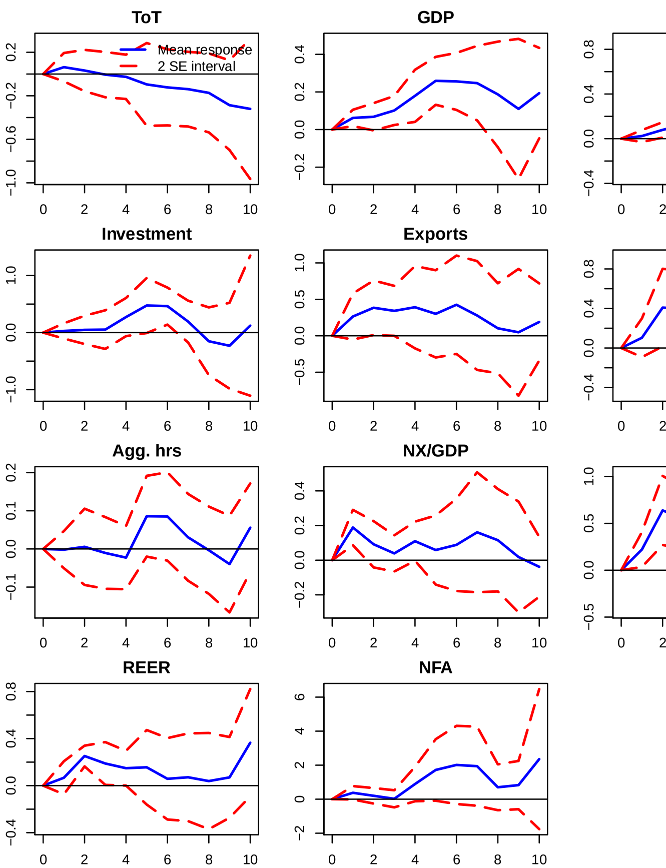
* $p < .1$, ** $p < .05$, *** $p < .01$

2.5 The role of international asset portfolios

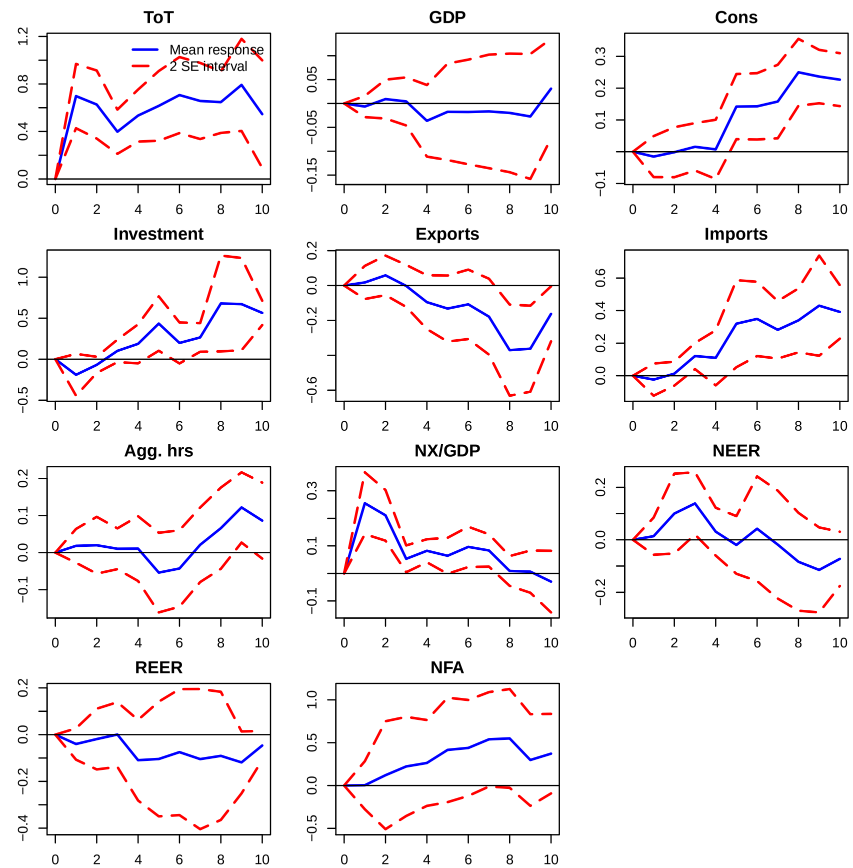
If our story is correct, one would expect that countries with large, positive net asset positions have larger income effects from terms-of-trade shocks, and therefore responses to terms-of-trade shocks which exhibit more clearly the features we emphasize.

As a test of our interpretation, therefore, we re-run our baseline regressions on sub-samples of countries where NFA positions are, on average, more versus less positive. Of course, this does not cleanly identify the marginal impact of a larger NFA position on the dynamic responses; many other factors which might impact the dynamic responses may also vary systematically with average NFA, confounding identification. However, this is at least a chance for the data to reject our theory. If sub-sample estimation provided results which were strongly at odds with our interpretation, then this would be of grave concern. Instead, this exercise shows that countries which have more positive NFA positions in fact feature responses where the income effect appears to dominate.

We exploit the heterogeneity in average net foreign asset positions in Figure 3, running our baseline regressions for countries with average NFA in the top and bottom quartiles. Countries in the bottom quartile have an NFA below XX% of GDP, and see textbook effects of a terms of trade shock: output expands, but the wealth effect is small, so consumption rises in line with output. Both exports and imports rise, although the effect is hard to distinguish from zero. However, countries in the top quartile have an average NFA above YY% of GDP, so their asset



(a) Bottom quartile of average NFA



(b) Top quartile of average NFA

Figure 3: Estimated Impulse Responses to a one percent commodity terms of trade shock, split by average NFA

portfolios appreciate considerably when their currency appreciates. These countries see a large wealth effect from a terms of trade shock: consumption increases significantly and and output does not; exports decline, and imports increase.

Countries do not appear to use their international portfolio exposure to nominal exchange rates to hedge against terms of trade shocks. Given that terms of trades improvements cause losses to portfolios biased towards the domestic currency, we expect that countries which are more exposed to terms of trade shocks would choose debt portfolios with less home bias. On the contrary, the relationship between home bias and terms of trade exposure is weak or even positively correlated. Figure 4 plots this correlation for several quantities, against the country's median NFDA home bias. First, median country openness – imports plus exports relative to GDP – has nearly zero correlation with home bias, but higher trade dependence increases an economy's exposure to terms of trade shocks *ceteris paribus.* Next, countries with a higher standard deviation of nominal exchange rate growth tend to have more home bias, despite the associated increase in portfolio exchange rate risk. Finally, countries with greater home bias also have higher terms of trade volatility, which is plotted for both the measured terms of trade as well as the constructed commodity terms of trade. These correlations are unconditional, omitting other variables that must be accounted for with a model, so in Section 5 we calculate the optimal debt portfolios implied by our model, and find further evidence supporting this puzzle.

2.6 The role of exchange rate flexibility

Exchange rates play an important role in our story. The income effect which we argue for stems from a valuation effect – that terms of trade shocks feature a contemporaneous exchange rate movement which increases net foreign assets. This suggests that countries with flexible exchange rates should be more likely to exhibit income-effect-like responses. As with average asset position, the choice of fixed versus flexible exchange rates may be correlated with other determinants of the dynamic responses. And so this evidence should also be treated as suggestive, rather than highly compelling.

We define observations as under a fixed exchange rate regime if they meet two tests. First, the

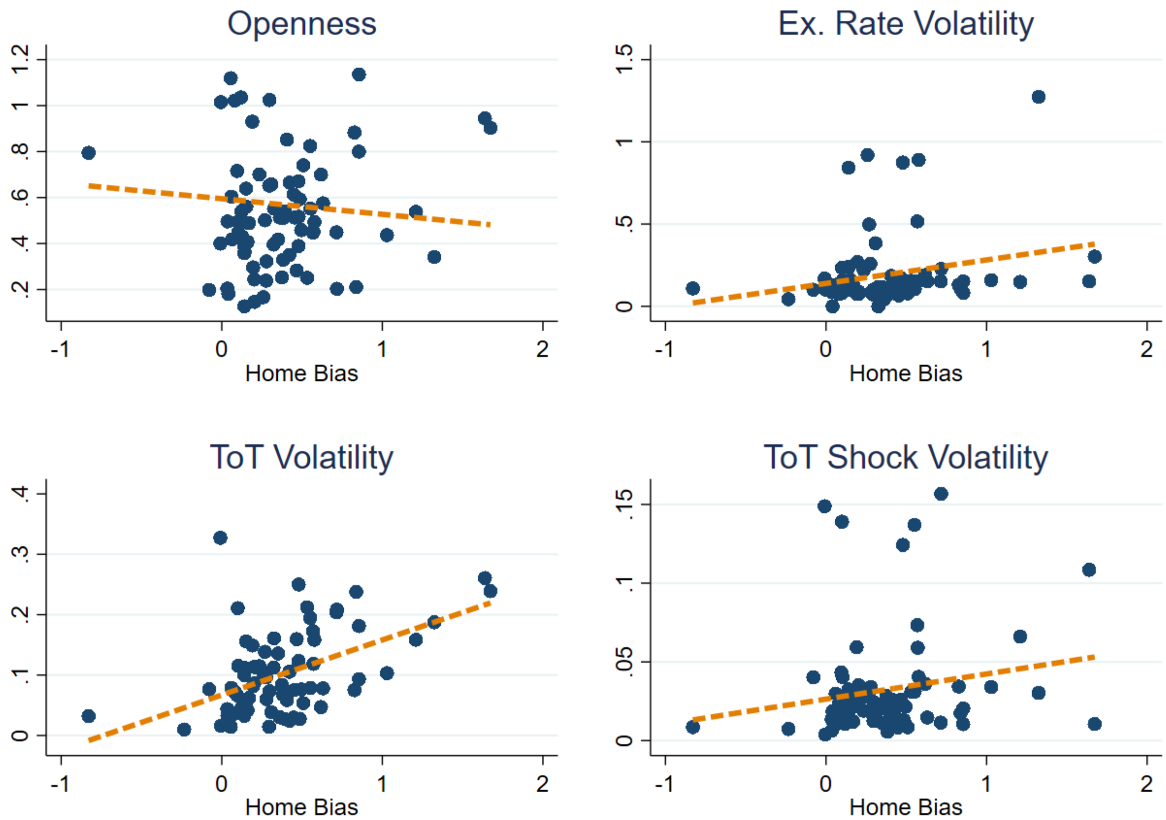
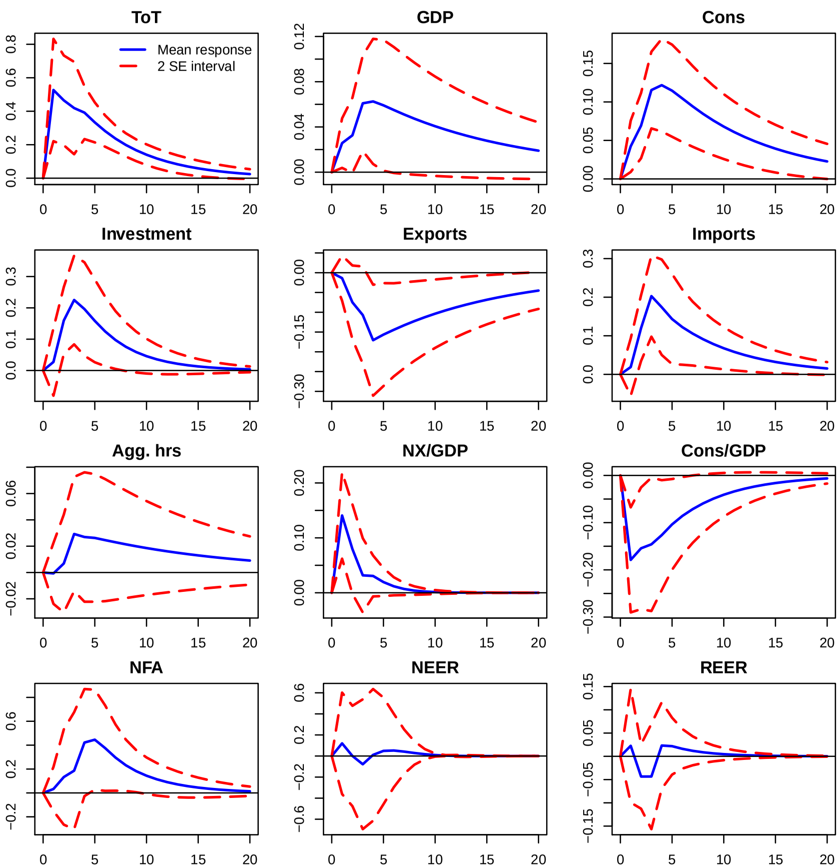


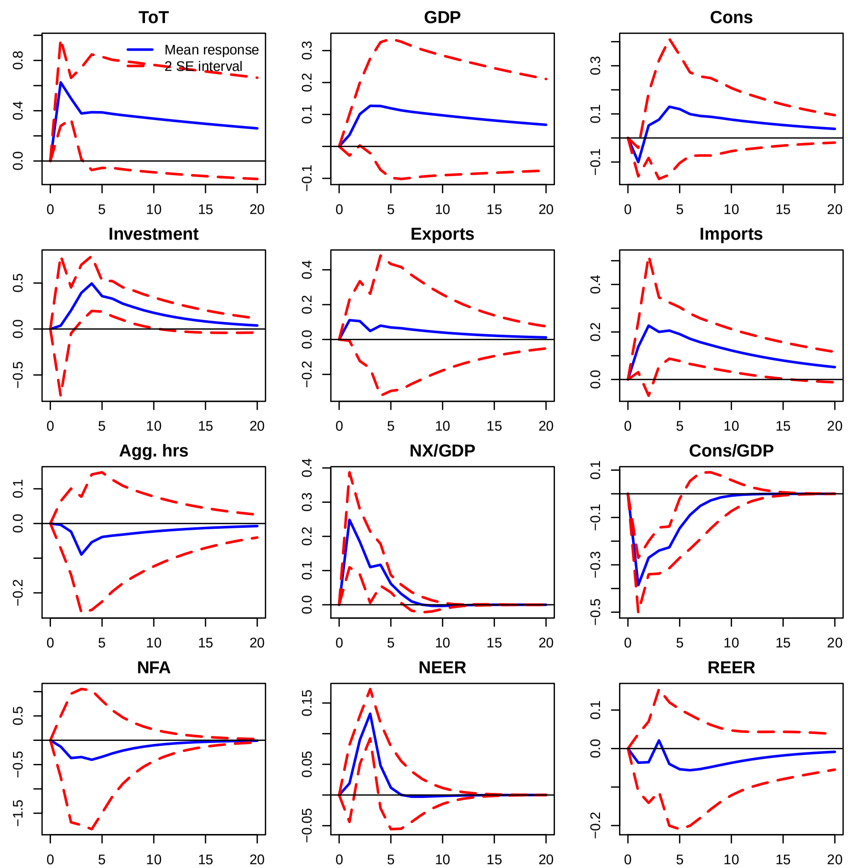
Figure 4: Countries' Domestic Currency Home Bias and Related Risks

country must be have a majority of periods where exchange rates are pegged, according to the Klein and Shambaugh (2010) classification.. Second, the observation itself must be one where the exchange rate in that country is pegged. For an observation to be classified as flexible, a similar two-part test must be met.

Figure 5 displays the impulse responses for subsamples of fixed or flexible exchange rate observations. When exchange rates are flexible consumption responses are more clearly identified, exports and imports move sooner in response to a shock, and NFA increases by more, and more persistently.



(a) Flexible exchange rates



(b) Pegged exchange rates

Figure 5: Estimated Impulse Responses to a one percent commodity terms of trade shock, split by exchange rate regime

3 A Small Open Economy Model with Gross Asset Positions

3.1 Households

The representative household in the small open economy maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{6}$$

where C_t is domestic consumption in period t, and L_t is labor hours in period t. The utility function is King and Rebelo (1999)'s Constant Frisch utility function,

$$U(C_t, L_t) = \frac{1}{1-\gamma} \left(\tilde{C}_t^{1-\gamma} \left(1 - (1-\gamma)\chi \frac{\epsilon}{1+\epsilon} L_t^{\frac{\epsilon+1}{\epsilon}} \right)^{\gamma} - 1 \right) \tag{7}$$

where ϵ is the Frisch labor supply elasticity and $\frac{1}{\gamma}$ is the intertemporal elasticity of substitution along the balanced growth path.[9]

The household earns wage income $W_{i,t}$ per hour of labor, denominated in domestic currency[10]. The price level is $P_{C,t}$. The household has access to three asset markets. It can hold non-contingent domestic bonds B_{t+1} at price $\frac{1}{R_{t+1}}$, which pays one unit of domestic currency in period $t+1$, and it can hold non-contingent foreign bonds B^F_{t+1} at price $\frac{1}{R^F_{t+1}}$, which pays one unit of a global foreign currency. Both of these bonds are in zero net supply.

The household can also hold domestic capital K_t. Capital is governed by the law of motion

$$K_{t+1} = I_t(1 - \frac{\varphi}{2}(\frac{I_t}{I_{t-1}} - 1)^2)\tau_{I,t} - \delta K_t \tag{8}$$

where the $\frac{\varphi}{2}(\frac{I_t}{I_{t-1}} - 1)^2$ is an investment adjustment cost, following the Christiano, Eichenbaum, and Evans (2005) formulation, and $\tau_{I,t}$ is a stochastic investment productivity shock. Households choose a utilization rate U_t for their capital, at cost κU_t^{ω} in units of the generic output good. Capital services $U_t K_t$ return the rental rate R^K_t.

[9] Trabandt and Uhlig (2011) prove that this is the unique utility function which features a constant Frisch elasticity, and a constant intertemporal elasticity of substitution.

[10] "Currency" here serves the role only of a unit of account, not a means of exchange nor a store of value. Given that we are most interested in wealth effects induced by relative fluctuations in competing units of account, this is an appropriate simplification for our purposes.

The household can spend its income on consumption C_t, investment in capital I_t, and financial assets for the next period. Therefore the household's period budget constraint (denominated in domestic currency) is

$$W_t L_t + R_t^K U_t K_t + B_t + \mathcal{E}_t B_t^F = P_t C_t + P_t I_t + P_t \kappa U_t^\omega + \frac{B_{t+1}}{R_{t+1}} + \mathcal{E}_t \frac{B_{t+1}^F}{R_{t+1}^F} \tag{9}$$

where $\mathcal{E}_t$ is the nominal exchange rate between the domestic and global foreign currencies, and P_t is the price level for both consumption and investment.

Households choose consumption, investment, labor, utilization, and assets to maximize their utility (6) subject to constraints (8) and (9). Optimality implies that their labor supply is determined by setting the real wage equal to the marginal rate of substitution between consumption and labor:

$$\frac{W_t}{P_t} = \tau_{L,t} \frac{\gamma \chi C_t L_t^{\frac{1}{\epsilon}}}{1 - (1-\gamma)\chi \frac{\epsilon}{1+\epsilon} L_t^{\frac{\epsilon+1}{\epsilon}}} \tag{10}$$

where $\tau_{L,t}$ is a stochastic labor wedge, which represents frictions in the labor market. The choice of capital utilization requires that the marginal real rental income be equal to the marginal cost:

$$\frac{R_{K,t}}{P_t} = \kappa \omega U_t^{\omega-1} \tag{11}$$

Households have three assets, which they price with three Euler equations. The Euler equation for domestic nominal bonds is

$$1 = R_{t+1} E_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{12}$$

where Λ_{t+1} is the household's stochastic discount factor in period $t+1$. The Euler equation for foreign nominal bonds is

$$1 = R_{t+1}^F E_t \left[\Lambda_{t+1} \frac{\mathcal{E}_{t+1} P_t}{\mathcal{E}_t P_{t+1}} \right] \tag{13}$$

and the Euler equation for capital is

$$Q_t = E_t \left[\Lambda_{t+1} \left(\frac{R_{K,t+1}}{P_{t+1}} + Q_{t+1}(1-\delta) \right) \right] \tag{14}$$

where Q_t is Tobin's Q, the cost of a marginal unit of capital in terms of output. Q_t is determined by the first order condition for investment, which is a dynamic equation:

$$1 = Q_t \tau_{I,t} \left(1 - \frac{\varphi}{2} (\frac{I_t}{I_{t-1}} - 1)^2 - \varphi (\frac{I_t}{I_{t-1}})^2 \right) + E_t \left[\Lambda_{t+1} Q_{t+1} \tau_{I,t+1} \varphi (\frac{I_{t+1}}{I_t})^3 \right] \tag{15}$$

Finally, the utility function implies that the stochastic discount factor is given by

$$\Lambda_{t+1} = \beta \frac{\tilde{C}_t^{\gamma}}{\tilde{C}_{t+1}^{\gamma}} \frac{\left(1-(1-\gamma)\chi\frac{\epsilon}{1+\epsilon}L_{t+1}^{\frac{\epsilon+1}{\epsilon}}\right)^{\gamma}}{\left(1-(1-\gamma)\chi\frac{\epsilon}{1+\epsilon}L_{t}^{\frac{\epsilon+1}{\epsilon}}\right)^{\gamma}} \tau_{EE,t} \tag{16}$$

We assume global demand for assets is characterized by two equations. The interest rate on international bonds is fixed at $R_t^F = \frac{1}{\beta}$, and uncovered interest rate parity holds:

$$R_t^F E_t[\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}] = R_t \tag{17}$$

3.2 Firms

There are four types of firms in each country: intermediate goods producers, intermediate assemblers, importers and final retailers.

The intermediate goods producers produce a variety of inputs, indexed by $i \in I$. These producers are monopolistic competitors, and use a Cobb-Douglas production function to combine labor $L_{i,t}$ and capital services $\tilde{K}_{i,t}$ into output $Y_{i,t}$ by

$$Y_{i,t} = A_t \tilde{K}_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \tag{18}$$

where A_t is the stochastic total factor productivity which is shared by all primary producers. Primary producers hire labor at wage rate W_t and hire capital services at rental rate R_t^K. Primary producers are monopolistic, so they choose their output price $P_{i,t}$. However, they face Calvo-style price stickiness, and can only update their price with probability λ.

The intermediate assemblers aggregate inputs into a homogeneous domestic output good Y_t^D by a CES production function:

$$Y_t^D = \left(\int_{i\in I} Y_{i,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{19}$$

where η is the elasticity of substitution. These firms are perfectly competitive, purchasing variety i at price $P_{i,t}$ and selling output at price P_t^D.

Importers aggregate homogeneous output goods $X_{j,t}$ from other countries, which are indexed by $j \in J$. Importing firms produce a homogeneous imported output good Y_t^M by a CES production function:

$$Y_t^M = \left(\int_{j\in J} \mu_{j,t} X_{j,t}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \tag{20}$$

where ζ is the elasticity of substitution, and $\mu_{j,t}$ is a stochastic demand shifter that satisfies $\int_{j\in J}\mu_{j,t}=1$. Importers are perfectly competitive, purchasing input j at price $P^M_{j,t}$ and selling output at price P^M_t.

Lastly, final retailers combine assembled domestic goods Y^D_t and aggregated imported goods Y^M_t into the final output good Y_t by a CES production function:

$$Y_t=\left(\theta^{\frac{1}{\xi}}(Y^D_t)^{\frac{\xi-1}{\xi}}+(1-\theta)^{\frac{1}{\xi}}(Y^M_t)^{\frac{\xi-1}{\xi}}\right)^{\frac{\xi}{\xi-1}} \tag{21}$$

where ξ is the elasticity of substitution and the share parameter θ captures the degree of home bias. Final retailers are perfectly competitive, and sell output at the aggregate price level P_t. Output goods are used for consumption C_t, investment I_t, government spending G_t, and exports X_t, implying the market clearing condition

$$Y_t=C_t+I_t+G_t+X_t \tag{22}$$

3.3 Prices

Profit maximization in the retail sector implies that demand for the domestic aggregate Y^D_t and imported aggregate Y^M_t are given by

$$(\theta\frac{Y_t}{Y^D_t})^{\frac{1}{\xi}}=\frac{P^D_t}{P_t}\qquad((1-\theta)\frac{Y_t}{Y^M_t})^{\frac{1}{\xi}}=\frac{P^M_t}{P_t} \tag{23}$$

Profit maximization in the import sector implies that demand for the imported bundle from country j is given by

$$(\mu_{j,t}\frac{Y^M_t}{X_{j,t}})^{\frac{1}{\zeta}}=\frac{P_{j,t}}{P^M_t} \tag{24}$$

The imported good Y^M_t is the same across countries so the law of one price holds:

$$P^M_t=\mathcal{E}_tP^M_{W,t} \tag{25}$$

where $P^M_{W,t}$ is the world price of the imported good, expressed in the global currency.

The export demand function holds in all countries as well, so the aggregate demand for exports X_t is

$$X_t=\mu_t\int_{j\in J}Y^M_{j,t}(\frac{\mathcal{E}_tP^M_{j,t}}{\mathcal{E}_{j,t}P_t})^{\zeta}$$

$P^M_{j,t}$ is the price of imports in country j's currency. $\mathcal{E}_t$ is the domestic country's exchange rate with the global currency, so $\mathcal{E}_t/\mathcal{E}_{j,t}$ is the exchange rate with country j. The law of one price (25) implies that this export demand equation can be written as

$$X_t = \mu_t(\frac{\mathcal{E}_t P^M_{W,t}}{P_t})^\zeta \int_{j \in J} Y^M_{j,t}$$

Then, by defining world trade as $Y^M_{W,t} \equiv \int_{j\in J} Y^M_{j,t}$ and the terms of trade as $S_t \equiv \frac{P_t}{\mathcal{E}_t P^M_{W,t}}$, the demand for exports is given by

$$\mu_t \frac{Y^M_{W,t}}{X_t} = S_t^\zeta \tag{26}$$

Profit maximization in the intermediate assembly sector implies that demand for the intermediate good of type i is given by

$$(\frac{Y^D_t}{Y_{i,t}})^{\frac{1}{\eta}} = \frac{P^D_t}{P_{i,t}} \tag{27}$$

Intermediate goods firms face a dynamic problem, because they can only update their prices stochastically. Appendix F characterizes this problem and shows that it leads to a standard linearized New Keynesian Philips Curve:

$$\log(\Pi^D_t) = \beta \log(\Pi^D_{t+1}) - \Theta \log(\frac{\eta(1-\alpha)P^D_t Y^D_t}{(\eta+1)W_t L_t}) \tag{28}$$

where $\Pi^D_t \equiv \frac{P^D_t}{P^D_{t-1}}$ is inflation of the domestic aggregate and $\Theta > 0$. This equation operates as the labor demand equation. The capital services demand can either be described by a similar dynamic equation, or equivalently by setting the marginal rate of transformation to the relative input price:

$$\frac{\alpha L_t}{(1-\alpha)\tilde{K}_t} = \frac{R^K_t}{W_t} \tag{29}$$

The aggregate price level P_t is determined by a cash constraint:

$$P_t Y_t = \nu M_t \tag{30}$$

where M_t is the exogenous stochastic money supply, and ν is the money velocity.

3.4 Exogenous Shocks

There are four exogenous shocks which we assume follow a stationary AR(1) process in logs. Money:

$$\log M_{t+1} = \rho_M \log M_t + \varepsilon_{M,t+1} \tag{31}$$

export demand:

$$\log \mu_{t+1} = \rho_\mu \log \mu_t + \varepsilon_{\mu,t+1} \tag{32}$$

the labor wedge:

$$\log \tau_{L,t+1} = \rho_L \log \tau_{L,t} + \varepsilon_{L,t+1} \tag{33}$$

and investment productivity:

$$\log \tau_{I,t+1} = \rho_I \log \tau_{I,t} + \varepsilon_{I,t+1} \tag{34}$$

We also assume that log productivity $\log A_t$ follows an AR(1) process with drift, as in Aguiar and Gopinath (2007):

$$\log A_{t+1} = \rho_A \log A_t + (1-\alpha) \log \Gamma_{t+1} + \varepsilon_{A,t+1} \tag{35}$$

where Γ_{t+1} is the trend component. The inclusion of labor share $1-\alpha$ as a coefficient on the trend implies that variables with a balanced growth rate can be stationarized by dividing by Γ_t. This trend is itself stochastic, following the process

$$\Gamma_{t+1} = e^{z_{t+1}} \Gamma_t$$

$$z_{t+1} = \rho_z z_t + (1-\rho_z)\bar{z} + \varepsilon_{z,t+1} \tag{36}$$

where $\bar{z}$ is the average growth rate.

3.5 Equilibrium Definition

A competitive equilibrium in this economy consists of sequences of 28 quantities: assets K_t, B_t, B_t^F; prices, $W_t, R_{K,t}, R_t, P_t, P_t^D, P_t^M, \mathcal{E}_t$; allocations, $Y_t, Y_t^D, Y_t^M, C_t, I_t, U_t, L_t, X_t$; and other quantities, $Q_t, \Lambda_t, S_t, \Pi_t^D$; and realizations of the exogenous stochastic states $A_t, z_t, M_t, \mu_t, \tau_{L,t}, \tau_{I,t}$, such that:

1. Households maximize their intertemporal utility subject to constraints, which implies that 10 equations must be satisfied: the capital law of motion (8); the budget constraint (9); 3 Euler equations: (12), (13), and (14); foreign demand for domestic assets (uncovered interest rate parity) (17); the cash constraint (30); and 3 first order conditions for labor (10), utilization (11), and investment (15).

2. Firms maximize profits, satisfying 9 equations: input demand by the intermediate producers (28) and (29); domestic and foreign input aggregation (21) and two demand functions in (23); global export demand (26); the law of one price (25); the resource constraint (22), and to a first order approximation output aggregates by

$$Y_t^D \approx A_t(U_t K_t)^\alpha L_t^{1-\alpha} \tag{37}$$

3. Three variable definitions must be satisfied: the definitions of the terms of trade, domestic producer inflation, and stochastic discount factor (16).

4. The exogenous stochastic states must satisfy the laws of motion (31)-(36).

4 Model Estimation and Results

4.1 Solutions Methods and Equilibrium Selection

To solve the model, we linearize around a deterministic steady state, and calculate equilibrium using Dynare (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot, 2011). Linearized small open economy models are not generally stable (Schmitt-Grohe and Uribe, 2003), so we stabilize the model by adding an ad hoc Endogenous Discount Factor (EDF). Specifically, we assume that instead of β, the household's discount factor is $\beta e^{\vartheta(\bar{C}-C_t)}$, where $\bar{C}$ is steady state consumption and $\vartheta > 0$. Moreover, we assume that households do not internalize the effect of C_t on the EDF, or equivalently that aggregate consumption enters the EDF instead of household consumption. Finally, we must select the steady state asset position, and we choose zero NFA.

Linear models with portfolio problems also suffer from multiplicity. In the linear approximation, households are risk neutral, and therefore indifferent between all potential asset allocations for a given level of wealth. To resolve this indeterminacy, we assume that countries choose an allocation of bonds that matches their empirical home bias. In Section 5 we compare this equilibrium with empirical portfolios to equilibrium where households choose the model's optimal portfolio, using a higher order solution method.

4.2 Calibration and Estimation

We estimate the model independently for each of 93 countries. We use annual observations on 12 variables covering the period 1972-2014, although the panel is unbalanced; data availability is reported in Table 6. 6 quantities are taken from the PWT national account data: GDP, consumption, investment, exports, imports, and hours. All quantities are expressed in real, per capita terms. We also construct 5 time series of prices from the PWT data: the consumption deflator which we treat as the price level, the GDP deflator which we treat as the price of domestically produced goods, the relative deflators of exports to imports which we take as treat as the terms of trade, the relative deflators of investment to consumption which we treat as an observation on Tobin's Q, and the US dollar exchange rate. Lastly, we take annual NFA positions from the Benetrix-Lane-Schambaugh dataset. All time series are expressed in per capita terms, and all time series (except NFA) are logged and then linearly detrended country-by-country.

We select some parameter values a priori, and estimate the remainder. Table 3 reports the chosen parameters. We endeavored to select standard values for small open economy models. The unique case is the goods home bias parameter θ, which we calibrate for each country to match its openness - the average ratio of exports plus imports to GDP. Table 4 reports the parameters that we estimate country-by-country with Bayesian methods. For elasticities, we choose a Gamma distribution with mean 2 and standard deviation $\sqrt{2}$; the Gamma distribution restricts the range to positive values, and this mean/variance combination implies a mode of one, so that we place equal probabilities on elasticities being greater or less than one. For the pricing and investment frictions, we choose an exponential distribution, so that the values are restricted to be positive, but also allowing for estimates near zero. Standard deviations for shocks and measurement errors are also exponential. Finally, we select a uniform prior on $[-1, 1]$ for the shock autocorrelations in order to exclude explosive shocks, but to be otherwise agnostic.

Parameter	Interpretation	Value
β	Discount rate	0.98
γ	Risk Aversion	2
δ	Depreciation rate on capital	0.1
α	Capital share	0.4
η	Intermediate elast.	5
ω	Utilitzation elast.	2
θ	Home bias	varies

Table 3: Calibrated Parameters

Parameter	Interpretation	Prior
ϵ	Frisch labor elasticity	Gamma(2,$\sqrt{2}$)
ζ	Substitution across imports	Gamma(2,$\sqrt{2}$)
ξ	Domestic/foreign subst.	Gamma(2,$\sqrt{2}$)
Θ	NK friction	Exp(1)
φ	Inv. adjustment cost	Exp(1)
ρ_A, ρ_M, ρ_μ, ρ_L, ρ_I, ρ_Z	Shock autocorrelations	Un(-1,1)
σ_A, σ_M, σ_μ, σ_L, σ_I, σ_Z	Shock std. dev.'s	Exp(1)
ς_Y, ς_I, ς_S, ς_P, ς_{NFA}	Measurement errors	Exp(1)

Table 4: Estimated Parameters

4.3 Response to a Terms of Trade Shock

Figure 6 plots the model response to a positive $\varepsilon_{\mu,t}$ shock, which increases foreign demand for domestic goods μ_t and improves the terms of trade. The impulse responses are plotted for two different calibrations. In the first calibration, the country has no home bias in bonds. In the second calibration, the country has home bias worth 100% of GDP, (i.e. a 1% domestic currency appreciation would appreciate the country's portfolio by 1% of GDP). All other parameters are set to the estimates for Mexico. The plotted values are log deviations (except for NFA) so the impulse responses can be interpreted as elasticities.

Output rises after a terms of trade improvement. Labor and capital utilization both fall, so this increase is final output is driven by increasing imports to use in production. The country takes advantage of its improved terms of trade by exporting more goods, temporarily reducing capital investment and consumption to do so. The resulting trade surplus leads to an accumulation of

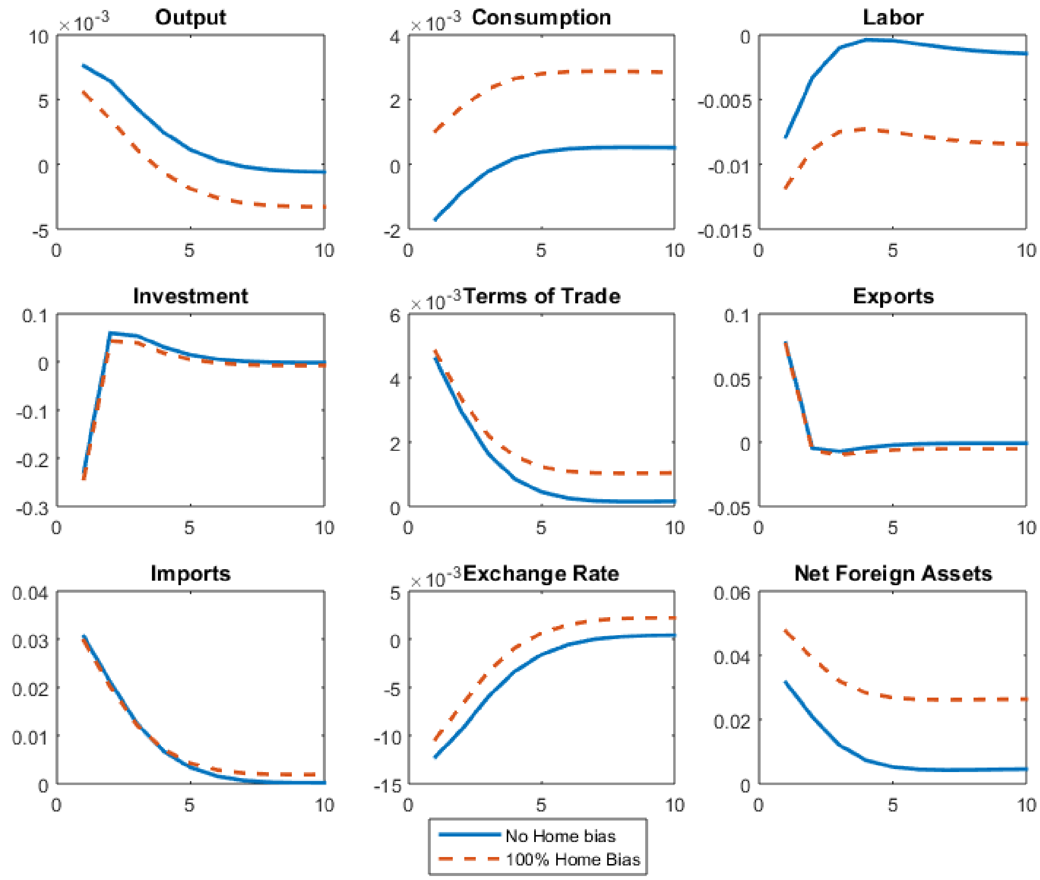


Figure 6: Equilibrium Impulse Responses to a Terms of Trade Shock

net foreign assets. The terms of trade slowly revert to the steady state, households begin to enjoy their increased wealth, increasing consumption, investment, and leisure.

If a country has a home-biased bond portfolio, it undergoes a more extreme response to the shock. The terms of trade improvement reduces the real exchange rate, because the domestic consumption bundle becomes more expensive relative to the foreign consumption bundle. The price level moves but not by enough to offset the real exchange rate, so the nominal exchange rate decreases (i.e. the currency appreciates) as well. If the country has home-biased portfolio bond portfolio, the exchange rate movement appreciates the country's net foreign asset position. This is increases household wealth, so they choose more consumption. Moreover, this consumption increase is long-lasting, and would be permanent if not for the endogenous discount factor. Households also choose to work less, which slightly decreases the output response.

How do these responses compare to the evidence from section 2? When countries hold home-

	Percent of Variance	Absolute Variance
Mean	3.46%	0.026
Median	2.83%	0.014
Minimum	0.02%	0.0008
Maximum	12.46 %	0.259
Standard Deviation	2.69%	0.039
Observations	70	70

Table 5: Contribution of Terms of Trade Shocks to Log Consumption Variance

biased portfolios, consumption rises by more than output. This is true in the data, but only when countries have flexible exchange rates. When exchange rates are pegged, there is no valuation effect from the shock, and dynamics resemble the zero home-biased response. When countries are home biased, aggregate hours fall as they do in the data, but when countries are not home biased, hours would rise as they do for countries with fixed exchange rates. In all cases, the responses of exchange rates and net foreign assets agree with the reduced form estimates. However two quantities rise immediately in the data – net exports and capital investment – while in the model they only rise after an immediate decline.

4.4 Importance of Terms of Trade Shocks

After estimating the model for each country, we decompose the variance of consumption into the contribution from each fundamental shock. The first column of Table 5 reports summary statistics for the percent of consumption variance that is driven by terms of trade shocks across countries. The second column reports summary statistics for the total consumption variance that is driven by terms of trade shocks. We can estimate the model for 70 countries, which is less than the 93 countries studied in Section 2.

For most countries, terms of trade shocks are a small component of consumption variance. Much more important are labor demand shocks and trend productivity shocks. But there is considerable heterogeneity across countries. While the average country sees terms of trade determine only 2-3% of their consumption variance, others are much larger, such as Zimbabwe (8.9%), Nepal (11.1%), and Nigeria (12.5%). What determines this heterogeneity?

Countries for whom terms of trade shocks are more important to the business cycle tend to have portfolios that are more home biased in their currency exposure. Figure 6 illuminates the channel: when countries receive terms of trade improvements, their consumption rises by more when they are more home biased. Figure 7 plots this correlation. Among countries with low home bias, the correlation is weak. But nearly all countries with a home bias above 75% of GDP feature a relatively large percentage of their consumption variance that is driven by terms of trade shocks.

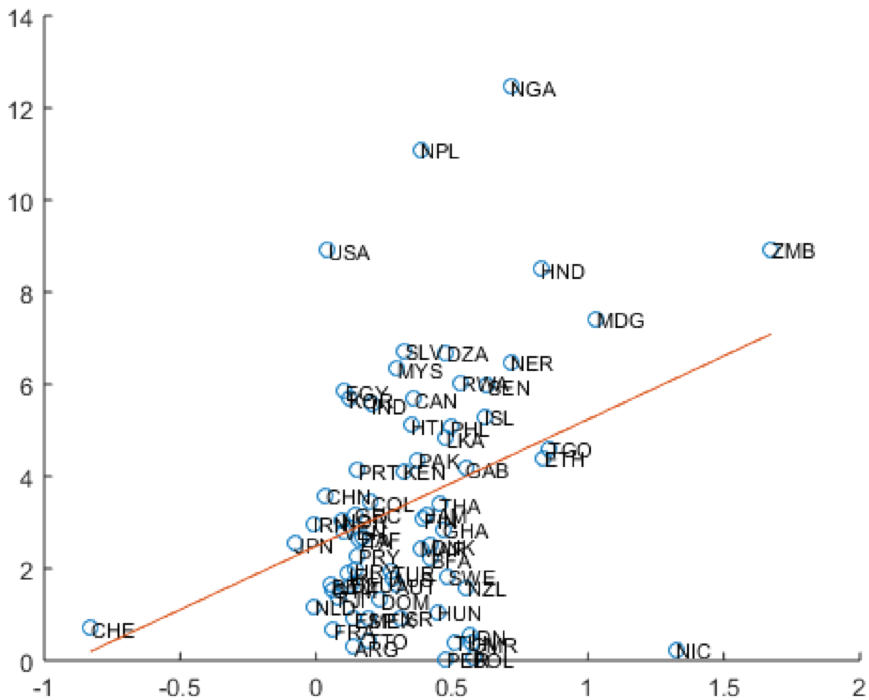


Figure 7: Portfolio Home Bias and Percent of Variance due to ToT Shocks

It could be that countries who are otherwise more exposed to terms of trade shocks choose a more home biased portfolio. So we control for the other country-specific parameters and still find that countries that are more home biased have greater contributions of terms of trade shocks to their consumption variance. Figure 8 shows this relationship, plotting the conditional correlation between portfolio home bias and the absolute log consumption variance that is due to terms of trade shocks. Why are countries with a greater consumption exposure to terms of trade shocks choosing financial portfolios that reinforce this exposure? We explore this question in the next section.

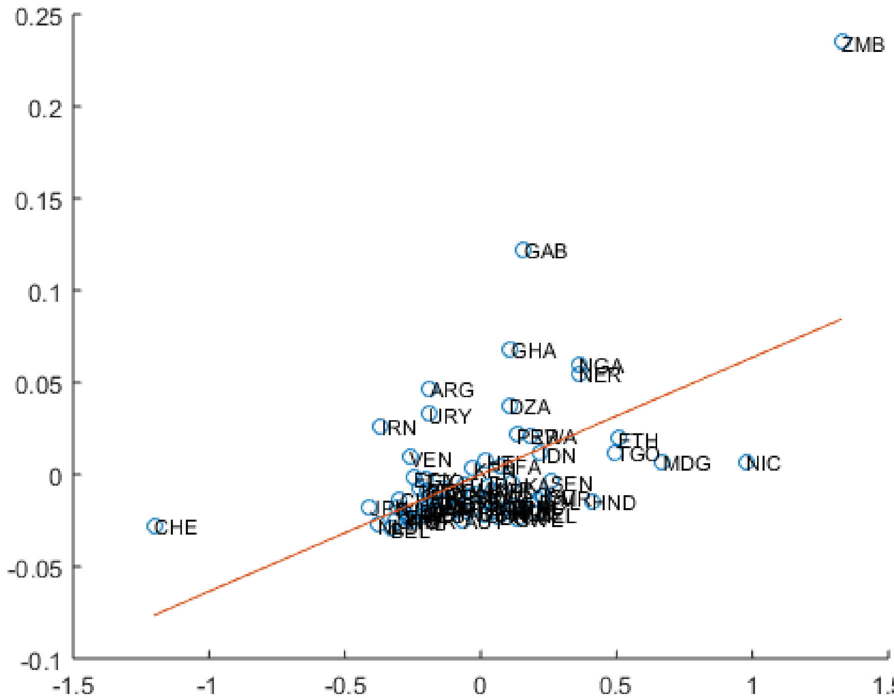


Figure 8: Portfolio Home Bias and Absolute Variance due to ToT Shocks (Conditional)

5 Optimal Portfolios

–IN PROGRESS–

In this section we use the method in Appendix C to calculate optimal country portfolios and compare versus observed portfolios.

5.1 Environment

We consider the dynamic problem of a household that chooses risky assets to hold, and may also earn some endowment income. The risky assets are elastically supplied, so the household's choices do not affect the assets' returns. We can also consider this to be the problem faced by a representative household in a small open economy with internationally traded assets.

Household maximize the present discounted utility flow of consumption. They have a power utility function

$$u(C_t) = \frac{C_t^{1-\gamma} - 1}{1 - \gamma}$$

where γ is the coefficient of relative risk aversion. Households discount future utility by β.

In every period, households face a budget constraint. They hold J different assets $A_{j,t}$ indexed by $j \in \{1, ...J\}$, each earning stochastic return $R_{j,t}$. Some of these assets are chosen endogenously (like stocks and bonds) while other exogenous assets encompass non-financial wealth, which pay returns as endowments (such as inelastic labor income). We treat the exogenous assets like Lucas (1978) trees: households can choose how much to hold, but holding the exogenous amount is imposed as an equilibrium condition, so that expected returns to these assets still satisfy an Euler Equation, which prices the asset.

We define ex-post wealth W_t as

$$W_t \equiv \sum_{j=1}^{J} R_{j,t} A_{j,t} \tag{38}$$

Households spend their wealth on consumption and next period's assets, so their budget constraint is

$$W_t = C_t + \sum_{j=1}^{J} A_{j,t+1}$$

The solution to the household's standard intertemporal maximization problem must satisfy an Euler equation for each endogenous asset A_j:

$$1 = \beta E_t \left[R_{j,t+1} \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \right] \tag{39}$$

5.2 Approximation

Next, we approximate the Euler equations around the model's ergodic mean (if it exists). Overbars denote quantities at the ergodic mean, e.g. $\overline{R}_j$. Lowercase bars denote deviations from the ergodic mean, but this definition differs. For most variable, we consider the log deviation, e.g.: $c_t \equiv \log C_t - \log \overline{C}$, but for returns we consider the proportional linear deviation: $r_{j,t} \equiv \frac{R_{j,t} - \overline{R}_j}{\overline{R}_j}$.

The first order approximation of the Euler equation 39 around the ergodic mean is

$$1 \approx E_t \left[\beta \overline{R}_j + \beta \overline{R}_j \gamma (c_t - c_{t+1}) + \beta \overline{R}_j r_{j,t+1} \right]$$

and the second order approximation is

$$1 \approx E_t \left[\beta \overline{R}_j + \beta \overline{R}_j \gamma (c_t - c_{t+1}) + \beta \overline{R}_j r_{j,t+1} + \beta \overline{R}_j \frac{\gamma^2}{2} (c_t - c_{t+1})^2 + \beta \overline{R}_j \gamma (c_t - c_{t+1}) r_{j,t+1} \right]$$

At the ergodic mean itself (i.e. where $c_t = 0$) the approximation reduces to

$$\text{[Ergodic Mean]:}\;\; 1 \approx E_t \left[\beta \overline{R}_j + \beta \overline{R}_j \frac{\gamma^2}{2} c_{t+1}^2 - \beta \overline{R}_j \gamma c_{t+1} r_{j,t+1} \right] \tag{40}$$

The first order approximation of the wealth definition (38) is

$$w_{t+1} \approx \sum_j \overline{S}_j (r_{j,t+1} + a_{j,t+1})$$

where $\overline{S}_j \equiv \frac{\overline{R}_j \overline{A}_j}{\overline{W}}$ is the share of ex-post wealth held in asset j. At the ergodic mean in period t, $a_{j,t+1} = 0$ because it is chosen in period t. The wealth equation becomes

$$\text{[Ergodic Mean]:}\;\; w_{t+1} \approx \sum_j \overline{S}_j r_{j,t+1} \tag{41}$$

A property of homothetic utility functions is that at the ergodic mean, consumption is proportional to wealth. When consumption and wealth are expressed in log deviations, this implies $c_t = w_t$. Combining this relationship at the ergodic mean with the wealth equation (41) and the Euler equation (40) yields

$$\text{[Ergodic Mean]:}\;\; 1 \approx E_t \left[\beta \overline{R}_j + \beta \overline{R}_j \frac{\gamma^2}{2} c_{t+1}^2 - \beta \overline{R}_j \gamma r_{j,t+1} \sum_{k=1}^{J} \overline{S}_k r_{k,t+1} \right]$$

and when the expectation of second-order terms is replaced with variances and covariances at the ergodic mean, we get

$$\text{[Ergodic Mean]:}\;\; \frac{1}{\beta \overline{R}_j} \approx 1 + \frac{\gamma^2}{2} Var(c) - \gamma \sum_{k=1}^{J} \overline{S}_k Cov(r_j, r_k) \tag{42}$$

5.3 General Solution for Average Portfolio Shares

There are J unknown average portfolio shares $\overline{S}_j$, J unknown average returns $\overline{R}_j$, but only $J+1$ equations that relate them: the J Euler equations plus an adding up equation $1 = \sum_{j=1}^{J} \overline{S}_j$. Accordingly, $J-1$ additional restrictions are necessary to determine a solution.

Express Euler equation (42) for asset j

$$\lambda_j \approx \sum_{k=1}^{J} \overline{S}_k Cov(r_j, r_k) \tag{43}$$

where $\lambda_j \equiv \frac{1}{\gamma} + \frac{\gamma}{2} Var(c) - \frac{1}{\gamma\beta\overline{R}_j}$ Stacking this equation for each asset j yields the matrix equation

$$\vec{\lambda} = \Sigma_r \vec{S} \tag{44}$$

where Σ_r is covariance matrix of returns at the ergodic mean, while $\vec{\lambda}$ and $\vec{S}$ are the vectors of λ_j and $\overline{S}_j$.

The necessary restrictions to solve for the portfolio may be imposed on shares or λ's in some combination. We consider linear restrictions to determine the portfolio. When imposed on shares, we express the supply restrictions as the matrix equation

$$\vec{\alpha}_S = \Theta_S \vec{S} \tag{45}$$

where the vector $\vec{\alpha}_S$ and the matrix Θ_S are known. One row of this matrix equation must include the adding-up constraint $1 = \sum_{j=1}^{J} \overline{S}_j$.

Similarly, there may be restrictions on the vector $\vec{\lambda}$, such as some assets having the same average return. When these restrictions are linear, they are expressed as

$$\vec{\alpha}_\lambda = \Theta_\lambda \vec{\lambda} \tag{46}$$

Finally, stack the restrictions so that $\vec{\alpha} \equiv \begin{pmatrix} \vec{\alpha}_S \\ \vec{\alpha}_\lambda \end{pmatrix}$ and $\Theta \equiv \begin{pmatrix} \Theta_S \\ \Theta_\lambda \Sigma_r \end{pmatrix}$.

Theorem 1 *If Θ is square and of rank J, then there is a unique set of average portfolio positions which is given by:*

$$\vec{S} = \Theta^{-1} \vec{\alpha} \tag{47}$$

Proof. Average portfolio positions must satisfy the matrix Euler equation 44, and restriction equations 45 and 46. Substitute 44 into 46 to get

$$\vec{\alpha}_\lambda = \Theta_\lambda \Sigma_r \vec{S}$$

Stack this equation with 45 to get

$$\begin{pmatrix} \vec{\alpha}_S \\ \vec{\alpha}_\lambda \end{pmatrix} = \begin{pmatrix} \Theta_S \vec{S} \\ \Theta_\lambda \Sigma_r \vec{S} \end{pmatrix}$$

which we can write as

$$= \vec{\alpha} = \Theta \vec{S}$$

The matrix Θ has J columns by definition. So if it is square and of rank J, it has a unique inverse and the average portfolio shares are given by

$$\vec{S} = \Theta^{-1} \vec{\alpha}$$

■

In the next section, we apply this solution method to a general set of assets for small open economies with a tractable restriction: symmetry across countries. In Appendix C we apply the solution method to several simpler examples.

5.4 Portfolio Shares When Open Economies Have Symmetric Assets

In this section we consider a special case where small open economies have symmetric assets. This gives some structure with which to impose restrictions. We show that portfolio shares in this case are given by the coefficients from a regression of nonfinancial returns on a set of financial returns. This generalizes the result of Coeurdacier and Gourinchas (2016) to a dynamic model with many types of assets.

Suppose that countries hold three types of assets: J_D domestic financial assets, J_F foreign financial assets, and a single nonfinancial asset. Each domestic asset has a symmetric foreign asset, e.g. domestic and foreign currency-denominated bonds, domestic and foreign equities, etc. Then $J_D = J_F$ and the total number of assets is $J = 1 + 2J_D$. Notationally, define $\vec{S}_D$ as the vector of domestic asset shares and $\vec{S}_F$ as the vector of symmetric foreign asset shares. We stack these as $\vec{S} \equiv \begin{pmatrix} \vec{S}_D \\ \vec{S}_F \\ S_N \end{pmatrix}$ where S_N is the share of nonfinancial income. Similarly, we stack the λ_j's as $\vec{\lambda} \equiv \begin{pmatrix} \vec{\lambda}_D \\ \vec{\lambda}_F \\ \lambda_N \end{pmatrix}$.

All financial assets are in zero net supply, imposing the restrictions

$$\vec{S}_D = -\vec{S}_F \tag{48}$$

Given that all financial shares are symmetric, the sum of domestic shares plus the sum of foreign shares will add to zero because

$$\sum_{j=1}^{J_D} S_j = -\sum_{j=1}^{J_F} S_j$$

so the adding up constraint $\sum_{j=1}^{J} S_j = 1$ implies that the nonfinancial asset will make up 100% of wealth, i.e. $S_N = 1$.

Similarly, all domestic financial assets pay the same average real return as their foreign counterpart, imposing the restrictions

$$\vec{\lambda}_D = \vec{\lambda}_F \tag{49}$$

As an example, if the symmetric assets are domestic and foreign nominal bonds, then the restriction implies that uncovered interest rate parity holds on average.

The nonfinancial asset is a bundle that may include real assets such as capital, as well as labor income or other nonfinancial income sources. Thus we can consider countries trading capital in this context by treating it as if countries always hold their own capital but trade equity or other claims in the capital stocks.

To find the solution to this portfolio problem, we first introduce some notation. Let $\vec{r} \equiv \begin{pmatrix} \vec{r}_D \\ \vec{r}_F \\ r_N \end{pmatrix}$ denote the vector of asset returns ordered in the same way as $\vec{S}$. Next, define Σ_{RR} as the variance matrix of relative returns $\vec{r}_D - \vec{r}_R$, and Σ_{RN} as the covariance matrix between relative returns and nonfinancial returns r_N.

Theorem 2 *If countries have symmetric assets such that each finanical asset has a paired asset with the opposite-signed average portfolio shares and the same average real return (Restrictions 48 and 49), and if the returns covariance matrix Σ_r is nonsingular, then the domestic average portfolio shares are given by*

$$\vec{S}_D = -\Sigma_{RR}^{-1}\Sigma_{RN} \tag{50}$$

Proof. Partition Σ_r, the covariance matrix for $\vec{r}$, by

$$\Sigma_r \equiv \begin{pmatrix} \Sigma_{DD} & \Sigma_{DF} & \Sigma_{DN} \\ \Sigma_{FD} & \Sigma_{FF} & \Sigma_{FN} \\ \Sigma_{ND} & \Sigma_{NF} & \Sigma_{NN} \end{pmatrix}$$

where $\Sigma_D D$ is the covariance matrix for $\vec{r}_D$, $\Sigma_F D$ is the covariance matrix for $\vec{r}_F$ and $\vec{r}_D$, and so forth.

The Euler matrix equation 44 can be rewritten with these partitions as

$$\begin{pmatrix} \vec{\lambda}_D \\ \vec{\lambda}_F \\ \lambda_N \end{pmatrix} = \begin{pmatrix} \Sigma_{DD} & \Sigma_{DF} & \Sigma_{DN} \\ \Sigma_{FD} & \Sigma_{FF} & \Sigma_{FN} \\ \Sigma_{ND} & \Sigma_{NF} & \Sigma_{NN} \end{pmatrix} \begin{pmatrix} \vec{S}_D \\ \vec{S}_F \\ S_N \end{pmatrix}$$

Restriction 49 then implies

$$\begin{pmatrix} \Sigma_{DD} & \Sigma_{DF} & \Sigma_{DN} \end{pmatrix} \begin{pmatrix} \vec{S}_D \\ \vec{S}_F \\ S_N \end{pmatrix} = \begin{pmatrix} \Sigma_{FD} & \Sigma_{FF} & \Sigma_{FN} \end{pmatrix} \begin{pmatrix} \vec{S}_D \\ \vec{S}_F \\ S_N \end{pmatrix}$$

Applying restriction 49 then yields the equation

$$\Sigma_{DD}\vec{S}_D - \Sigma_{DF}\vec{S}_D + \Sigma_{DN}S_N = \Sigma_{FD}\vec{S}_D - \Sigma_{FF}\vec{S}_D + \Sigma_{FN}S_N$$

The adding up equation implies $S_N = 1$. Applying this and collecting coefficients yields

$$\left(\Sigma_{DD} - \Sigma_{DF} - \Sigma_{FD} + \Sigma_{FF}\right) S_D + \Sigma_{DN} - \Sigma_{FN} = 0$$

We can substitute with the covariance matrices for relative returns $\vec{r}_D - \vec{r}_R$, using the following relationships:

$$\Sigma_{DD} - \Sigma_{DF} - \Sigma_{FD} + \Sigma_{FF} = \Sigma_R \qquad \Sigma_{DN} - \Sigma_{FN} = \Sigma_{RN}$$

Then our equation becomes

$$\Sigma_R S_D + \Sigma_{RN} = 0$$

Finally, given the premise that Σ_R is nonsingular, it can be inverted to yield the result:

$$\vec{S}_D = -\Sigma_{RR}^{-1}\Sigma_{RN}$$

■

6 Testing Country Portfolios

In this section we compare countries' optimal international portoflios with their obserseved portfolios and find XX YY and ZZ. (IN PROGRESS)

6.1 Optimal Portfolios with Nominal Bonds and Nonfinancial Wealth

In this section we find the optimal portfolio for an economy with asset structure considered in the model of Section 3. Countries earn income from domestic labor and capital, and trade noncontingent nominal bonds denominated in domestic and foreign currencies. In the model, countries are unable to trade capital internationally, but we also consider an alternative version in which capital is tradable.

The household's three assets are domestic currency bonds $B_{D,t}$, foreign currency bonds $B_{F,t}$, and non-financial income, which has present value $A_{N,t}$. The standard Euler equations with power utility for each of the bonds are:

$$1 = \beta R_{D,t} E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \frac{P_t}{P_{t+1}} \right]$$

$$1 = \beta R_{F,t} E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right]$$

where P_t is the domestic price level, $\mathcal{E}_t$ is the exchange rate, $R_{D,t}$ is the risk-free return on domestic bonds, and $R_{F,t}$ is the risk-free return on foreign bonds. The Euler equation pricing non-financial wealth A_N is

$$1 = \beta E_t \left[R_{N,t} \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \right]$$

where $R_{N,t}$ denotes the real return on nonfinancial wealth.

Let $\tilde{R}_{D,t+1} \equiv R_{D,t} \frac{P_t}{P_{t+1}}$ and $\tilde{R}_{F,t+1} \equiv R_{F,t} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$ denote the real returns on domestic and foreign nominal bonds. Expressed in the log deviations, the real returns become

$$\tilde{r}_{D,t+1} = r_{D,t} + p_t - p_{t+1}$$

$$\tilde{r}_{F,t+1} = r_{F,t} + p_t - p_{t+1} + e_{t+1} - e_t$$

where $r_{D,t}$, $r_{F,t}$, p_t, and e_t are respectively the log deviations of the domestic interest rate, foreign interest rate, price level and exchange rate.

The relative log return on domestic and foreign bonds is denoted by $r_{R,t+1} \equiv \tilde{r}_{D,t+1} - \tilde{r}_{F,t+1}$, which is given by

$$r_{R,t+1} = r_{D,t} - r_{F,t} + e_t - e_{t+1}$$

That is, the relative return on domestic bonds is the interest rate differential less the exchange rate appreciation.

There are three assets, so we must make two restrictions to find the average portfolio shares using Theorem 1. First, we assume that domestic bonds are globally in zero net supply, so that the average share of domestic bonds $\overline{S}_D = -\overleftarrow{S}_F$. Second, we assume the UIP holds at the ergodic mean, so that the bonds face the same average real return $\overline{\tilde{R}}_D = \overline{\tilde{R}}_F$. These restrictions are necessary to apply Theorem 2.

Applied to this environment with two nominal bonds a nonfinancial asset, Theorem 2 gives the average share of domestic bonds as

$$\overline{S}_D = -\frac{Cov(r_{R,t+1}, r_{N,t+1})}{Var(r_{R,t+1})} = \frac{Cov(e_{t+1} - e_t + r_{F,t} - r_{D,t}, r_{N,t+1})}{Var(e_{t+1} - e_t + r_{F,t} - r_{D,t})} \tag{51}$$

6.2 Calculating Returns in the Data

In this section, we describe how we construct a cross-country panel of relative bond returns and nonfinancial returns.

Relative bond returns are straight forward. We frame our relative returns with the United States as the reference country. Thus exchange rates e_t are relative to USD and the interest rate differential $r_{D,t} - r_{F,t}$ is relative to US interest rates. We take exchange rates and interest rates on bank deposits from the IMF's IFS database.

We treat a country's GDP per capita as its nonfinancial income, so the country's nonfinancial asset $A_{N,t}$ is the present value of future GDP per capita. With this specification, capital income is included in nonfinancial income. However, this assumption does not preclude a country from trading capital; it can effectively sell capital by holding negative claims on its capital. Calculating the return to the nonfinancial asset requires some further assumptions, which we detail in Appendix D. The method estimates nonfinacial returns country by country and uses real GDP per capita and the consumption price deflator from the Penn World Tables National Accounts data, and interest rates from the IFS database.

In the model, countries are unable to trade capital, but for robustness, we also examine the optimal portfolios when households are also able to trade claims on capital income. We define the gross return on domestic capital income $\tilde{R}_{DK,t+1}$ by

$$\tilde{R}_{DK,t+1} \equiv \frac{1}{Q_t}\left(\frac{R_{K,t+1}}{P_{t+1}} + Q_{t+1}(1-\delta)\right) \tag{52}$$

where Q_t is the relative price of consumption and investment goods. To illustrate how this definition ties to the model, this return is discounted in Euler Equation (14). Log linearized, the gross return on domestic capital income becomes

$$\tilde{r}_{DK,t+1} = \beta\left(\overline{\frac{R_K}{QP}}(r_{K,t+1} - p_{t+1}) + \overline{G_Q}(1-\delta)q_{t+1}\right) - q_t \tag{53}$$

Q_t is typically nonstationary, so this log-linearization is instead written with the assumption that it has a stationary trend, with average growth factor $\overline{G_Q}$. $\overline{\frac{R_K}{QP}}$ is the average real capital income, deflated by the investment goods price. We estimate $\overline{\frac{R_K}{QP}}$ and $\overline{G_Q}$ country by country. Finally, the relative price on domestic versus foreign capital is

$$\tilde{r}_{K,t} \equiv \tilde{r}_{DK,t} - \tilde{r}_{FK,t}$$

and we again use the United States' series as the foreign returns $\tilde{r}_{FK,t}$ to be consistent with the specification for bonds.

6.3 Implied Portfolios

7 Conclusion

In this paper we estimated the effects of terms of trade shocks on macroeconomies using a panel of commodity terms of trade indices. We found that a terms of trade improvement resembles a wealth increase, raising consumption by more than output, increasing the net foreign asset position, and decreasing net exports.

We hypothesized that the wealth increase is driven by a valuation effect on countries' external asset positions. Most countries have asset positions that increase in value when the country's currency appreciates. As evidence for our hypothesized channel, we show that a terms of trade improvement leads to a currency appreciation. Furthermore, we find that the puzzling wealth effects occur in countries with flexible exchange rates but not in countries with fixed exchange rates.

Next, we estimated a small open economy model for our sample of countries using their observed external asset positions and found considerable heterogeneity in the importance of terms of trade shocks for business cycles. In particular, we found that economies with portfolios that are more exposed to exchange rate valuation effects are also more sensitive to terms of trade shocks. This finding raises questions for future research. Why do countries choose an external asset position that increases or reinforces their sensitivity to terms of trade shocks? Are these countries behaving suboptimally? Are there frictions that constrain their ability to hold a different asset portfolio? Or are they using their observed exchange rate exposures to hedge against other shocks? We will explore these questions in future research.

References

Adams, J., and P. Barrett (2017): "Why are Countries' Asset Portfolios Exposed to Nominal Exchange Rates?," *IMF Working Paper*.

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2011): "Dynare Reference Manual, version 4," .

Aguiar, M., and G. Gopinath (2007): "Emerging Market Business Cycles: The Cycle Is the Trend," *Journal of Political Economy*, 115(1), 69–102.

Amano, R. A., and S. van Norden (1995): "Terms of trade and real exchange rates: the Canadian evidence," *Journal of International Money and Finance*, 14(1), 83–104.

Benetrix, A. S., P. R. Lane, and J. C. Shambaugh (2015): "International currency exposures, valuation effects and the global financial crisis," *Journal of International Economics*, 96(Supplement 1), S98–S109.

Bet, G., and C. Peluffo (2017): "Weak Institutions as a Barrier to Economic Growth," *Working Paper*.

Broda, C. (2001): "Coping with Terms of Trade Shocks: Pegs versus Floats," *American Economic Review*, 91(2), 376–380.

——— (2004): "Terms of trade and exchange rate regimes in developing countries," *Journal of International Economics*, 63(1), 31–58.

Chen, Y.-c., and K. Rogoff (2003): "Commodity currencies," *Journal of International Economics*, 60(1), 133–160.

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Coeurdacier, N., and P.-O. Gourinchas (2016): "When bonds matter: Home bias in goods and assets," *Journal of Monetary Economics*, 82(Supplement C), 119–137.

Drechsel, T., and S. Tenreyro (2017): "Commodity booms and busts in emerging economies," *Journal of International Economics*.

Feenstra, R. C., R. Inklaar, and M. P. Timmer (2015): "The next generation of the Penn World Table," *The American Economic Review*, 105(10), 3150–3182.

Fernandez, A., A. Gonzalez, and D. Rodriguez (2018): "Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies," *Journal of International Economics*, 111, 99–121.

Fernandez, A., S. Schmitt-Groh, and M. Uribe (2017): "World shocks, world prices, and business cycles: An empirical investigation," *Journal of International Economics*, 108, S2–S14.

Harberger, A. C. (1950): "Currency Depreciation, Income, and the Balance of Trade," *Journal of Political Economy*, 58(1), 47–60.

IMF (2018): "International Financial Statistics," .

Jorda, O. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95(1), 161–182.

King, R. G., and S. T. Rebelo (1999): "Chapter 14 Resuscitating real business cycles," *Handbook of Macroeconomics*, 1, 927–1007.

Klein, M. W., and J. C. Shambaugh (2010): *Exchange Rate Regimes in the Modern Era*. MIT Press.

Kose, M. A. (2002): "Explaining business cycles in small open economies: How much do world prices matter?," *Journal of International Economics*, 56(2), 299–327.

Laursen, S., and L. A. Metzler (1950): "Flexible Exchange Rates and the Theory of Employment," *The Review of Economics and Statistics*, 32(4), 281–299.

Lucas, R. E. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–1445.

Mendoza, E. G. (1995): "The Terms of Trade, the Real Exchange Rate, and Economic Fluctuations," *International Economic Review*, 36(1), 101–137.

Rogoff, K. (1996): "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 34(2), 647–668.

Schmitt-Groh, S., and M. Uribe (2018): "How Important Are Terms-of-Trade Shocks?," *International Economic Review*, pp. n/a–n/a.

Schmitt-Grohe, S., and M. Uribe (2003): "Closing small open economy models," *Journal of International Economics*, 61(1), 163–185.

Schmitt-Grohe, S., and M. Uribe (2017): "How important are terms of trade shocks?," *International Economic Review*.

Shousha, S. (2015): "Macroeconomic Effects of Commodity Booms and Busts," *manuscript, Columbia University*.

Trabandt, M., and H. Uhlig (2011): "The Laffer curve revisited," *Journal of Monetary Economics*, 58(4), 305–327.

World Bank, T. (2018): "Commodity Markets Outlook Database," .

Zeev, N. B., E. Pappa, and A. Vicondoa (2017): "Emerging economies business cycles: The role of commodity terms of trade news," *Journal of International Economics*, 108, 368–376.

A Data

A.1 Sample

Table 6 lists the countries an coverage of the four main data sources used above: the commodity terms of trade index, the Penn World Tables terms of trade index, national accounts data, and the Benetrix, Lane, and Shambaugh (2015) data on international asset holdings. And Table 7 lists the commodity price sub-indices used to construct the commodity terms of trade indices

Country	Commodity ToT	PWT ToT	National accounts	BLS asset data
Afghanistan	1972-2017	2013-2017	1972-2014	
Albania	1972-2017	2001-2017	1972-2014	1994-2012
Algeria	1972-2017	1981-2017	1972-2014	1990-2012
Andorra	1972-2017	2000-2015	1972-2014	
Angola	1972-2017	2014-2016	1972-2014	
Anguilla	1972-2017	2005-2005	1972-2014	
Antigua and Barbuda	1972-2017	2014-2017	1972-2014	
Argentina	1972-2017	1985-2017	1972-2014	1990-2012
Armenia	1972-2017	2004-2017	1990-2014	1998-2012
Aruba	1972-2017	2005-2017	1972-2014	
Australia	1972-2017	1984-2017	1972-2014	1990-2012
Austria	1972-2017	1983-2017	1972-2014	1990-2012
Azerbaijan	1972-2017	2001-2017	1990-2014	1996-2012
Bahamas	1972-2017	2002-2016	1972-2014	
Bahrain	1972-2017	2005-2017	1972-2014	
Bangladesh	1972-2017	1982-2014	1972-2014	1990-2012
Barbados	1972-2017	1985-2017	1972-2014	
Belarus	1972-2017	2003-2017	1990-2014	1996-2012
Belgium	1972-2017	1983-2017	1972-2014	1990-2012
Belize	1972-2017	1997-2017	1972-2014	
Benin	1972-2017	1997-2017	1972-2014	1990-2012
Bermuda	1972-2017	2017-2017	1972-2014	
Bhutan	1972-2017	2010-2013	1972-2014	
Bolivia	1972-2017	1982-2017	1972-2014	1990-2012
Bosnia Herzegovina	1972-2017	2008-2017	1990-2014	1999-2012
Botswana	1972-2017	2005-2017	1972-2014	1990-2012
Brazil	1972-2017	1988-2017	1972-2014	1995-2012
Brunei Darussalam	1972-2017	1991-2017	1972-2014	
Bulgaria	1972-2017	2001-2017	1972-2014	
Burkina Faso	1972-2017	1981-2017	1972-2014	1990-2012
Burundi	1972-2017	1998-2017	1972-2014	
Cabo Verde	1972-2017	2000-2017	1972-2014	
Cambodia	1972-2017	2005-2017	1972-2014	1993-2012
Cameroon	1972-2017	1981-2017	1972-2014	1990-2012

Canada	1972-2017	1983-2017	1972-2014	1990-2012
Cote d'Ivoire	1972-2017	2000-2016	1972-2014	1990-2012
Cayman Isds	1972-2017		1972-2014	
Central African Rep.	1972-2017	1981-2017	1972-2014	
Chile	1972-2017	1988-2017	1972-2014	1990-2012
China	1972-2017	1989-2017	1972-2014	1990-2012
China, Hong Kong SAR	1972-2017	1983-2017	1972-2014	1990-2012
China, Macao SAR	1972-2017	1981-2013	1972-2014	
Colombia	1972-2017	1983-2017	1972-2014	1990-2012
Comoros	1972-2017	2000-2014	1972-2014	
Congo, Rep.	1972-2017	2012-2015	1972-2014	1990-2012
Cook Isds	1972-2017	2006-2006	1972-2014	
Costa Rica	1972-2017	1991-2017	1972-2014	
Croatia	1972-2017	1997-2017	1990-2014	1998-2012
Cuba	1972-2017	2004-2007	1972-2014	
Cyprus	1972-2017	1981-2017	1972-2014	
Czech Republic	1972-2017	1998-2017	1990-2014	1994-2012
Czechoslovakia	1972-2017	1987-1991		
Denmark	1972-2017	1981-2017	1972-2014	1990-2012
Djibouti	1972-2017	1991-1993	1972-2014	
Dominica	1972-2017	1990-2011	1972-2014	
Dominican Republic	1972-2017	2006-2017	1972-2014	1990-2012
Ecuador	1972-2017	1985-2017	1972-2014	
Egypt	1972-2017	1986-2017	1972-2014	1990-2012
El Salvador	1972-2017	1991-2017	1972-2014	1990-2012
Eritrea	1972-2017		1990-2014	
Estonia	1972-2017	2000-2016	1990-2014	1993-2012
Ethiopia	1972-2017	1981-2017	1972-2014	1993-2012
Faeroe Isds	1972-2017	1981-2010		
Fiji	1972-2017	1982-2017	1972-2014	1990-2012
Finland	1972-2017	1981-2017	1972-2014	1990-2012
France	1972-2017	1983-2017	1972-2014	1990-2012
French Guiana	1972-2017	1981-1996		
French Polynesia	1972-2017	1981-2016	1972-2014	
FS Micronesia	1972-2017	2007-2014	1972-2014	
Gabon	1972-2017	1981-2010	1972-2014	1990-2012
Gambia. The	1972-2017	2000-2015	1972-2014	
Georgia	1972-2017	2001-2017	1990-2014	1996-2012
Germany	1972-2017	1983-2017	1972-2014	1990-2012
Ghana	1972-2017	2001-2014	1972-2014	1990-2012
Greece	1972-2017	1981-2017	1972-2014	1990-2012
Greenland	1972-2017	1981-2017	1972-2014	
Grenada	1972-2017	1989-2009	1972-2014	
Guadeloupe	1972-2017	1981-1996		
Guatemala	1972-2017	1991-2017	1972-2014	1990-2012

Guinea	1972-2017	2000-2009	1972-2014	1990-2012
Guinea-Bissau	1972-2017		1972-2014	
Guyana	1972-2017	2002-2017	1972-2014	
Haiti	1972-2017		1972-2014	1990-2012
Honduras	1972-2017	1991-2008	1972-2014	1990-2012
Hungary	1972-2017	1981-2017	1972-2014	1990-2012
Iceland	1972-2017	1982-2017	1972-2014	1990-2012
India	1972-2017	1988-2017	1972-2014	1990-2012
Indonesia	1972-2017	1984-2017	1972-2014	1990-2012
Iran	1972-2017	2002-2007	1972-2014	1994-2012
Iraq	1972-2017		1972-2014	
Ireland	1972-2017	1981-2017	1972-2014	1990-2012
Israel	1972-2017	1986-2017	1972-2014	1990-2012
Italy	1972-2017	1982-2017	1972-2014	1990-2012
Jamaica	1972-2017	1984-2017	1972-2014	1990-2012
Japan	1972-2017	1981-2017	1972-2014	1990-2012
Jordan	1972-2017	1986-2017	1972-2014	1990-2012
Kazakhstan	1972-2017	2000-2017	1990-2014	1996-2012
Kenya	1972-2017	1985-2011	1972-2014	1990-2012
Kiribati	1972-2017	1988-2017	1972-2014	
Kuwait	1972-2017	1992-2005	1972-2014	
Kyrgyzstan	1972-2017	2003-2014	1990-2014	1996-2012
Lao People's Dem. Rep.	1972-2017	2015-2017	1972-2014	
Latvia	1972-2017	1999-2017	1990-2014	1993-2012
Lebanon	1972-2017	2002-2015	1972-2014	
Lesotho	1972-2017	2005-2013	1972-2014	
Liberia	1972-2017	1983-1985	1972-2014	
Libya	1972-2017	1988-1992	1972-2014	
Lithuania	1972-2017	1999-2017	1990-2014	1993-2012
Luxembourg	1972-2017	2004-2017	1972-2014	
Madagascar	1972-2017	1981-2017	1972-2014	1990-2012
Malawi	1972-2017	1982-2016	1972-2014	1995-2012
Malaysia	1972-2017	1983-2017	1972-2014	1990-2012
Maldives	1972-2017	2000-2017	1972-2014	
Mali	1972-2017	2001-2009	1972-2014	1990-2012
Malta	1972-2017	1995-2017	1972-2014	
Martinique	1972-2017	1981-1996		
Mauritania	1972-2017	2005-2015	1972-2014	
Mauritius	1972-2017	1985-2017	1972-2014	
Mayotte	1972-2017	2005-2010		
Mexico	1972-2017	1991-2017	1972-2014	1990-2012
Mongolia	1972-2017	2001-2008	1972-2014	
Montenegro	1972-2017	2011-2017	1990-2014	
Montserrat	1972-2017	2004-2010	1972-2014	
Morocco	1972-2017	1981-2017	1972-2014	1990-2012

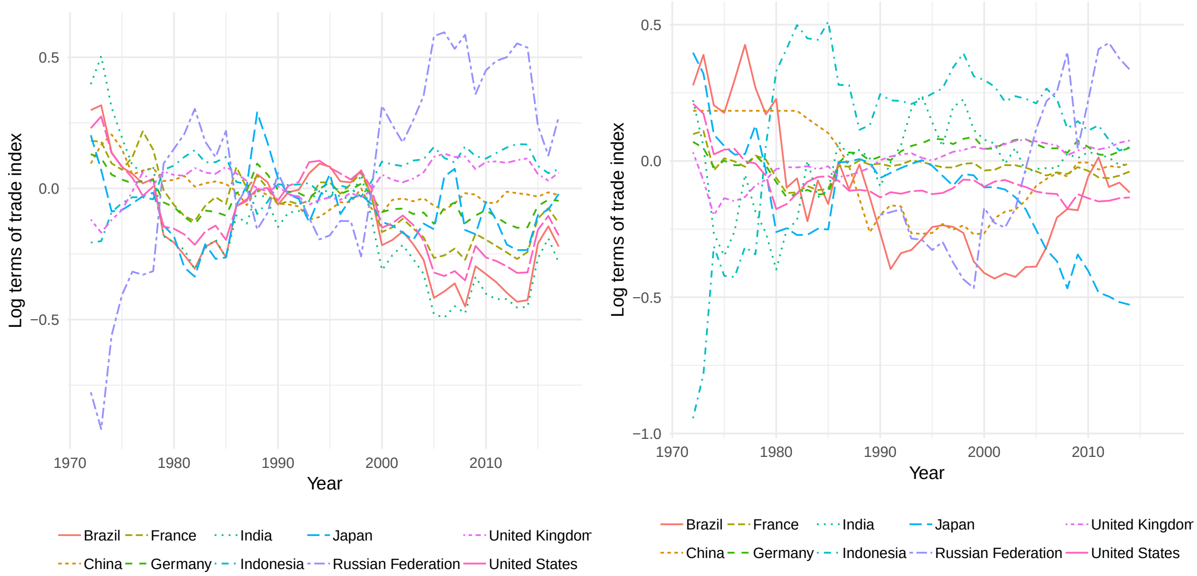
Mozambique	1972-2017	2005-2017	1972-2014	1990-2012
Myanmar	1972-2017	2015-2017	1972-2014	
Namibia	1972-2017	2005-2017	1972-2014	
Nepal	1972-2017	1987-2017	1972-2014	1990-2012
Neth. Antilles	1972-2017		1972-2012	
Netherlands	1972-2017	1983-2017	1972-2014	1990-2012
New Caledonia	1972-2017	1981-2016	1972-2014	
New Zealand	1972-2017	1984-2017	1972-2014	1990-2012
Nicaragua	1972-2017	1993-2017	1972-2014	1992-2012
Niger	1972-2017	1981-2017	1972-2014	1990-2012
Nigeria	1972-2017	2001-2015	1972-2014	1990-2012
Niue	1972-2017			
Norway	1972-2017	1981-2017	1972-2014	1990-2012
Oman	1972-2017	1986-2017	1972-2014	1990-2012
Pakistan	1972-2017	1987-2017	1972-2014	1990-2012
Palau	1972-2017		1972-2014	
Panama	1972-2017	1991-2017	1972-2014	
Papua New Guinea	1972-2017	1986-2005	1972-2014	1990-2012
Paraguay	1972-2017	1988-2017	1972-2014	1990-2012
Peru	1972-2017	1981-2017	1972-2014	1990-2012
Philippines	1972-2017	1982-2017	1972-2014	1990-2012
Poland	1972-2017	1985-2017	1972-2014	1990-2012
Portugal	1972-2017	1984-2017	1972-2014	1990-2012
Qatar	1972-2017	1994-2017	1972-2014	
Runion	1972-2017	1981-1996		
Korea	1972-2017	1981-2017	1972-2014	1990-2012
Moldova	1972-2017	1999-2017	1990-2014	1996-2012
Romania	1972-2017	1994-2017	1972-2014	1992-2012
Russian Federation	1972-2017	2001-2017	1990-2014	1996-2012
Rwanda	1972-2017	2006-2017	1972-2014	1990-2012
Saint Kitts and Nevis	1972-2017	1998-2017	1972-2014	
Saint Lucia	1972-2017	1990-2017	1972-2014	
Saint Pierre and Miquelon	1972-2017	1984-1985		
Saint Vincent and the Grenadines	1972-2017	2002-2013	1972-2014	
Samoa	1972-2017	1981-2017	1972-2014	
Sao Tome and Principe	1972-2017	2004-2017	1972-2014	
Saudi Arabia	1972-2017	1981-2017	1972-2014	
Senegal	1972-2017	1982-2017	1972-2014	1990-2012
Serbia	1972-2017	2010-2017	1990-2014	
Serbia and Montenegro	1972-2017	2001-2003		
Seychelles	1972-2017	1985-2017	1972-2014	
Sierra Leone	1972-2017		1972-2014	
Singapore	1972-2017	1984-2017	1972-2014	1990-2012
Slovakia	1972-2017	1999-2017	1990-2014	1994-2012
Slovenia	1972-2017	1997-2017	1990-2014	1993-2012

Solomon Isds	1972-2017	1989-2017	1972-2014	
Somalia	1972-2017		1972-2014	
South Africa	1972-2017	1981-2017	1972-2014	1990-2012
Spain	1972-2017	1983-2017	1972-2014	1990-2012
Sri Lanka	1972-2017	1984-2017	1972-2014	1990-2012
State of Palestine	1972-2017	2005-2017	1972-2014	
Sudan	1972-2017	2000-2013	1972-2014	
Suriname	1972-2017	1993-2015	1972-2014	
Swaziland	1972-2017	2005-2008	1972-2014	
Sweden	1972-2017	1981-2017	1972-2014	1990-2012
Switzerland	1972-2017	1981-2017	1972-2014	1990-2012
Syrian Arab Republic	1972-2017	1982-2011	1972-2014	1990-2012
Tajikistan	1972-2017		1990-2014	
TFYR of Macedonia	1972-2017	1999-2017	1990-2014	
Thailand	1972-2017	1981-2017	1972-2014	1990-2012
Timor-Leste	1972-2017		1990-2014	
Togo	1972-2017	1981-2017	1972-2014	1990-2012
Tonga	1972-2017	2005-2015	1972-2014	
Trinidad and Tobago	1972-2017	1984-2016	1972-2014	1990-2012
Tunisia	1972-2017	1985-2017	1972-2014	1990-2012
Turkey	1972-2017	1990-2017	1972-2014	1990-2012
Turkmenistan	1972-2017		1990-2014	1997-2012
Turks and Caicos Isds	1972-2017	2004-2010	1972-2014	
Tuvalu	1972-2017		1972-2014	
Uganda	1972-2017	1999-2017	1972-2014	1990-2012
Ukraine	1972-2017	2001-2017	1990-2014	1995-2012
United Arab Emirates	1972-2017	1993-2017	1972-2014	
United Kingdom	1972-2017	1983-2017	1972-2014	1990-2012
Tanzania	1972-2017	2002-2017	1972-2014	1990-2012
Uruguay	1972-2017	1988-2017	1972-2014	1990-2012
United States	1972-2017	1983-2017	1972-2014	1990-2012
Vanuatu	1972-2017	1985-1985	1972-2014	
Venezuela	1972-2017	1987-2014	1972-2014	1990-2012
Vietnam	1972-2017	2002-2017	1972-2014	1995-2012
Zambia	1972-2017	2000-2016	1972-2014	1993-2012
Zimbabwe	1972-2017	1995-2017	1972-2014	
No. countries	205	191	195	113

Table 6: Sample years. “Commodity ToT” is our commodity terms of trade index; “PWT ToT” is the terms of trade taken from the Penn World Tables; “National accounts” covers the macroeconomic variables we use; and “BLS asset data” is the Benetrix-Lane-Shambaugh data on international portfolios

Commodity Name	Matched SITC Code (2nd Revision)	Coverage
Aluminum	684	1960-2017
Bananas (US)	0573	1960-2017
Barley	0430	1960-2017
Beef	0111	1960-2017
Coal (Australian)	322	1970-2017
Cocoa	072	1960-2017
Coconut Oil	4243	1960-2017
Coffee (Arabica)	071	1960-2017
Copper	682	1960-2017
Copra	2231	1960-2017
Cotton	263	1960-2017
Crude Oil (Brent/Dubai/West Average)	333	1960-2017
Fish Meal	3501	1979-2017
Gold	9710	1960-2017
Ground Nut Oil	4234	1960-2017
Ground Nuts	2221	1980-2017
Iron Ore	281	1960-2017
Lead	685	1960-2017
Logs (Malaysian)	247	1960-2017
Maize	0440	1960-2017
Meat: Chicken	0114	1960-2017
Meat: Sheep	0112	1971-2017
Natural Gas (Index)	34	1960-2017
Nickel	683	1960-2017
Oranges	0571	1960-2017
Palm Oil	4242	1960-2017
Platinum	6812	1960-2017
Rice (Thai)	042	1960-2017
Rubber	23	1960-2017
Sawnwood (Malaysian)	248	1960-2017
Shrimp (Mexican)	0360	1960-2017
Silver	6811	1960-2017
Sorghum	04592	1960-2017
Soybean Oil	4232	1960-2017
Soybeans	2222	1960-2017
Sugar	061	1960-2017
Tea	0741	1960-2017
Tin	687	1960-2017
Tobacco	121	1960-2017
Wheat (US)	041	1960-2017
Zinc	686	1960-2017

Table 7: Disaggregated Commodities



(a) Commodities only (arithmetic aggregation)

(b) Full series

Figure 9: Terms of trade series for eight largest economies (log)

B Regression details

B.1 Shock estimation

We create terms of trade shocks Z_{it} by estimating country-specific autoregressive processes for the commodity-based index. The shocks Z_{it} are the estimated innovations. We follow this approach to extract the unpredictable component of fluctuations in the terms of trade index.

Figure 9 shows the terms of trade index as well as the estimated innovations for the twelve largest countries at the end of the sample period according to the Penn World Tables GDP expenditure measure. These countries account for nearly two thirds of world output at the end of the sample. Unsurprisingly, the largest innovations in the index correlate closely with well-known movements in commodity prices, especially the oil price which rose markedly in 1974, 1979, and 1999-2000, and fell in 1986 and 1998.

Using the Akiake Information Criterion, we find that the optimal lag length is one for 92 of the 96 countries[11]. For simplicity we therefore impose an AR(1) structure on all countries. Table 8 reports the distribution of persistence parameters.

[11] The exceptions are Burkina Faso (AIC lag length two), South Africa and Myanmar (both three), and Sudan (eight!).

B.2 Calculation of standard errors for dynamic objects

The impulse response for $X_{i,t}$ is given by equation (2), which we restate here for convenience

$$\Delta\hat{X}_{t+k} = \alpha + \gamma_k \Delta Z_t + \sum_{j=1}^{\min(N,k)} \beta_j \Delta\hat{X}_{t+k-j}$$

Denoting by $\mathbf{y} = (\gamma_1, \ldots, \gamma_M, \beta_1, \ldots, \beta_N)'$ the model parameter vector, then the variance of $\Delta\hat{X}_{t+k}$ can be calculated by:

$$var\Delta\hat{X}_{t+k} = \left(\frac{\partial\Delta\hat{X}_{t+k}}{\partial\mathbf{y}}\right)' \Sigma \left(\frac{\partial\Delta\hat{X}_{t+k}}{\partial\mathbf{y}}\right)$$

Where Σ is the variance-covariance matrix of $\mathbf{y}$. Given the dynamic nature of the model, the elements of the derivative w.r.t. the parameters is best expressed recursively, as:

$$\frac{\partial\Delta\hat{X}_{t+k}}{\partial\gamma_i} = 1_{k=i} + \sum_{j=1}^{\min(N,k)} \frac{\partial\Delta\hat{X}_{t+k-j}}{\partial\gamma_i}$$

$$\frac{\partial\Delta\hat{X}_{t+k}}{\partial\beta_i} = \Delta\hat{X}_{t+k-i} + \sum_{j=1}^{\min(N,k)} \frac{\partial\Delta\hat{X}_{t+k-j}}{\partial\beta_i}$$

The standard errors of the TDE cited in Table **??** can be computed similarly, by:

$$varTDE_X = \left(\frac{\partial\, TDE_X}{\partial\mathbf{y}}\right)' \Sigma \left(\frac{\partial\, TDE_X}{\partial\mathbf{y}}\right)$$

B.3 Robustness to alternative specifications

We consider six alternative approaches to estimating the dynamic effect of terms of trade shocks: first, we examine alternative specifications for the commodity index, second we consider sensitivity to income-based subsamples; [third, other values for M and N; fourth, joint pairwise estimation of equations defining key ratios; fifth, estimation without imposing the autoregressive specification on the shocks; and sixth, a GMM approach.

B.3.1 Alternative Indices

HERE we show baseline results for the following alternative indices: geometric aggregation, disaggregated commodities, full-sample shares, 3-year shares, and 5-year quantity weights.

B.3.2 Income-based subsamples

We wish understand the extent to which our empirical work holds across different groups of countries. We divide the countries into three commonly-used income groups[12]. Table 9 lists the division of the countries.

Total dynamic effects for the full sample and income-based categories are shown in Table 10. Point estimates exhibit some variation across this subsamples, but the same broad pattern holds in the subsamples as in the full sample. In all subsamples, consumption responses are larger than output, investment and imports increase, exports decline, and total labor input is close to zero.

In general, statistical significance is not as strong in the subsamples as in the the full sample. However, this is largely a function of the subsamples just being smaller. The sample size effect alone should increase standard errors by about 90% for advanced and low income developing economies and around 50% for emerging markets[13]. A cursory examination of Table 10 will reveal that estimates for the emerging and low income developing countries roughly respect these boundaries. Suggesting that loss of significance for these categories is a function of reduced sample size. However, the standard errors for advanced economies are much larger than this, suggesting that this is a much more heterogeneous subsample. Nevertheless, statistical significance of many of the key variables remains, albeit at lower significance levels.

[12]Further subdivision of the sample hampers makes estimation unreliable because the model includes time fixed effects. Time fixed effects are essentially cross-country averages, so these become very poorly estimated with small subsamples.

[13]The central limit theorem means that standard errors converge in proportion to rate $(\sqrt{n})^{-1}$. As there are 27 each of advanced, then if the effect of the reduction of size on the standard errors for this group relative to the whole sample should be approximately $\sqrt{96/27} = 1.88$, or around a 90% increase. Scale factors for the emerging and low income categories can be computed similarly: they contain 42 and 27 countries respectively.

B.3.3 Alternative lag structures

B.3.4 Estimation without autoregressive assumptions

B.3.5 Pairwise joint estimation of key equations

B.3.6 A generalized method of moments approach

C Calculating Average Portfolio Positions: Simple Examples

In this Appendix we use the method of Section 5 to solve for the average portfolio positions of several simple examples.

C.1 Common Average Returns

When the expected return at the ergodic mean $\overline{R}_j$ is common for all assets j, then the portfolio calculation is simplified considerably, because the average portfolio can be found without knowing the utility parameters β and γ.

Suppose that $\overline{R}_j = \overline{R}$ for all j. Then the Euler equation (42) becomes

$$\frac{1}{\gamma} + \frac{\gamma}{2}Var(c) - \frac{1}{\gamma\beta\overline{\overline{R}}} \approx \sum_{k=1}^{J} \overline{S}_k Cov(r_j, r_k) \tag{54}$$

The left hand side of this equation is common for all assets j, which we denote as $\lambda \equiv \frac{1}{\gamma} + \frac{\gamma}{2}Var(c) - \frac{1}{\gamma\beta\overline{\overline{R}}}$. Then stacking equation (54) for each asset j yields the matrix equation

$$\lambda\vec{1} = \Sigma_r \vec{S}$$

where Σ_r is covariance matrix of returns at the ergodic mean. This matrix is invertible, so the portfolio shares satisfy

$$\lambda\Sigma_r^{-1}\vec{1} = \vec{S} \tag{55}$$

Finally, the adding up equation $1 = \sum_{j=1}^{J} \overline{S}_j$ pins down the level of λ. In matrix form, this equation is $1 = \vec{1}'\vec{S}$. Subsituting for the shares with equation (55) gives an expression for lambda:

$$\lambda = \frac{1}{\vec{1}'\Sigma_r^{-1}\vec{1}}$$

Therefore portfolio shares are given by

$$\frac{\Sigma_r^{-1}\vec{1}}{\vec{1}'\Sigma_r^{-1}\vec{1}} = \vec{S} \tag{56}$$

C.1.1 Two risky assets

When there are only two risk assets (with the same average return) the average portfolio solution in equation (56) is straightforward and intuitive. With two assets A_1 and A_2, the Euler equations (54) become

$$\lambda = S_1 Var(r_1) + S_2 Cov(r_1, r_2)$$

$$\lambda = S_1 Cov(r_1, r_2) + S_2 Var(r_2)$$

Setting these equations equal and imposing the adding up equation $S_1 + S_2 = 1$ gives a single equation with a single unknown:

$$S_1 Var(r_1) + (1 - S_1) Cov(r_1, r_2) = S_1 Cov(r_1, r_2) + (1 - S_1) Var(r_2)$$

which yields the expression for the each asset's share of wealth::

$$S_1 = \frac{Var(r_2) - Cov(r_1, r_2)}{Var(r_1) - 2Cov(r_1, r_2) + Var(r_2)}$$

$$S_2 = \frac{Var(r_1) - Cov(r_2, r_1)}{Var(r_2) - 2Cov(r_2, r_1) + Var(r_1)}$$

Intuitively, the riskier an asset is, the lower is its share in the portfolio.

C.1.2 Example: Two nominal bonds

In this example, we consider a small open economy which holds two assets: it can hold a non-contingent bond denominated in domestic currency B_H, and a non-contingent bond denominated in foreign currency B_F.

The standard Euler equations for each of the bonds are:

$$1 = \beta R_H E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \frac{P_t}{P_{t+1}} \right]$$

$$1 = \beta R_F E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right]$$

where P_t is the domestic price level and $\mathcal{E}_t$ is the exchange rate. Accordingly, the log deviation of real returns for these assets are

$$r_{H,t+1} = p_t - p_{t+1}$$

$$r_{F,t+1} = p_t - p_{t+1} + e_{t+1} - e_t$$

where p_t and e_t are the log deviations of the price level and exchange rate respectively.

With common average returns (assuming UIP holds at the ergodic mean), the Euler Equations (54) become

$$\lambda = S_H Var(r_H) + S_F Cov(r_H, r_F)$$

$$\lambda = S_H Cov(r_H, r_F) + S_F Var(r_F)$$

Setting these equations equal and imposing the adding up equation $S_H + S_F = 1$ gives a single equation with a single unknown:

$$S_H Var(r_H) + (1 - S_H) Cov(r_H, r_F) = S_H Cov(r_H, r_F) + (1 - S_H) Var(r_F)$$

which yields the expression for the domestic bonds' share of wealth S_H:

$$S_H = \frac{Var(r_F) - Cov(r_H, r_F)}{Var(r_H) - 2Cov(r_H, r_F) + Var(r_F)}$$

This equation holds generally for any two risky assets r_H and r_F, but the definitions above imply $Var(r_H) = Var(p)$, $Var(r_F) = Var(p) + Var(e) + 2Cov(p,e)$, and $Cov(r_H, r_F) = Var(p) + Cov(p,e)$. Substituting with these expressions yields

$$S_H = \frac{Var(e) + Cov(p,e)}{Var(e)}$$

$$S_F = \frac{-Cov(p,e)}{Var(e)}$$

We can conclude that if exchange rate growth correlates with inflation (i.e. $Cov(p,e) > 0$), the country will go short foreign bonds ($S_F < 0$) in order to go long domestic bonds. However if the correlation were reversed ($Cov(p,e) < 0$), then the country will hold positive quantities of foreign bonds.

C.2 Heterogeneous Average Returns

When average returns differ across asset classes, restrictions on the supply of assets are necessary to find a solution. In these examples, we rely on the restriction that contracts are in zero net supply across symmetric small open economies.

C.2.1 Example: Two nominal bonds and nonfinancial income

In this example, we consider a small open economy which holds two financial assets: it can hold a non-contingent bond denominated in domestic currency B_H, and a non-contingent bond denominated in foreign currency B_F. It also earns stochastic non-financial income in every period. Denote the present value of their non-financial income as A_N, which pays real return R_N.

The standard Euler equations for each of the bonds are:

$$1 = \beta R_H E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \frac{P_t}{P_{t+1}} \right]$$

$$1 = \beta R_F E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right]$$

where P_t is the domestic price level and $\mathcal{E}_t$ is the exchange rate. The Euler equation pricing non-financial wealth A_N is

$$1 = \beta E_t \left[R_N \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \right]$$

Again, the log deviation of real returns for the nominal bonds are

$$r_{H,t+1} = p_t - p_{t+1}$$

$$r_{F,t+1} = p_t - p_{t+1} + e_{t+1} - e_t$$

where p_t and e_t are the log deviations of the price level and exchange rate respectively. The log deviation of real non-financial returns is denoted by $r_{N,t+1}$.

The three Euler Equations as written in (54) become

$$\lambda_H = S_H Var(r_H) + S_F Cov(r_H, r_F) + S_N Cov(r_H, r_N)$$

$$\lambda_F = S_H Cov(r_H, r_F) + S_F Var(r_F) + S_N Cov(r_F, r_N)$$

$$\lambda_N = S_H Cov(r_H, r_N) + S_F Cov(r_F, r_N) + S_N Var(r_N)$$

With 6 unknowns and 3 equations, 3 restrictions are required. As always, one restriction is that the shares sum to one. For the second restriction, we can impose that UIP holds at the ergodic mean, so that $\lambda_H = \lambda_F$, as in the two bond example. Finally, we might impose that bonds are in zero net supply around the world an countries are symmetric, which we can formalize as the third restriction: $S_N = 1$.

Imposing the restrictions and performing some algebra yields the share of wealth for domestic bonds:

$$S_H = \frac{Cov(r_F, r_N) - Cov(r_H, r_N)}{Var(r_H) + Var(r_F) - 2Cov(r_H, r_F)}$$

If the real return of foreign bonds covaries more with non-financial returns than domestic bonds, then countries will hold a positive share of home bonds as a hedge on non-financial income risk. Because $S_F = -S_H$ in this example, they will hold a negative share of foreign bonds, yielding home bias in nominal bonds.

Substituting for bond returns with exchange rates and prices yields a simpler expression for the domestic share:

$$S_H = \frac{Cov(\Delta e, r_N)}{Var(\Delta e)}$$

Therefore, if exchange rate growth Δe covaries with non-financial returns, then countries will choose a home-biased bond portfolio.

C.2.2 Example: Two nominal bonds, two equities, and nonfinancial income

TBD (easy, maybe redundant to section 5)

D Estimating Returns

THIS APPENDIX NEEDS CLEANING UP!!!

Consider the household's present value equation, where Y_t is GDP per capita and $\mathcal{M}_{t+1} \equiv \beta\left(\frac{C_t}{C_{t+1}}\right)^{\gamma}$ is the stochastic discount factor:

$$A_{N,t} = E_t\left[\mathcal{M}_{t+1}\left(Y_{t+1} + A_{N,t+1}\right)\right] \tag{57}$$

A log linear approximation gives

$$\overline{A}_N a_{N,t} = E_t\left[\overline{\mathcal{M}}((\overline{Y} + \overline{A}_N)m_{t+1} + \overline{Y}y_t + \overline{A}_N a_{N,t+1})\right] \tag{58}$$

Then use $\overline{\mathcal{M}} = \beta$, $\overline{A}_N = \overline{\mathcal{M}}(\overline{Y} + \overline{A}_N)$, and the recursive relationship to find

$$a_{N,t} = E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{\overline{Y}}{\overline{A}_N} y_{t+j} + m_{t+j} \right) \right] \tag{59}$$

The log-linearized Euler Equation for the domestic nominal bond is

$$0 = r_{D,t} + E_t \left[m_{t+1} + p_t - p_{t+1} \right]$$

Using the notation for the real return $\tilde{r}_{D,t+1}$ and substituting into EQN XXXX gives

$$a_{N,t} = E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{\overline{Y}}{\overline{A}_N} y_{t+j} - \tilde{r}_{D,t+j} \right) \right] \tag{60}$$

Next, we assume an AR(1) process for the object $\frac{\overline{Y}}{\overline{A}_N} y_t - \tilde{r}_{D,t}$, such that

$$\frac{\overline{Y}}{\overline{A}_N} y_{t+1} - \tilde{r}_{D,t+1} = \rho\left(\frac{\overline{Y}}{\overline{A}_N} y_t - \tilde{r}_{D,t}\right) + \varepsilon_{t+1} \tag{61}$$

Then equation YYYY can be rewritten as

$$a_{N,t} = \frac{\rho\beta}{1-\rho\beta} \left(\frac{\overline{Y}}{\overline{A}_N} y_t - \tilde{r}_{D,t} \right) \tag{62}$$

Lastly, we can define the return on nonfinancial wealth as

$$R_{N,t+1} \equiv \frac{Y_{t+1} + A_{N,t+1}}{A_{N,t}} \tag{63}$$

Log-linearized, this becomes

$$\overline{A}_N \overline{R}_N \left(a_{N,t} + r_{N,t+1} \right) = \overline{Y} y_{t+1} + \overline{A}_N a_{N,t+1}$$

and rearranged

$$a_{N,t} + r_{N,t+1} = \frac{\overline{Y}}{\overline{A}_N \overline{R}_N} y_{t+1} + \frac{1}{\overline{R}_N} a_{N,t+1}$$

and imposing $\overline{R}_N = 1/\beta$

$$a_{N,t} + r_{N,t+1} = \beta \frac{\overline{Y}}{\overline{A}_N} y_{t+1} + \beta a_{N,t+1}$$

Then, use the recursive relationship to get

$$r_{N,t+1} = -E_t\left[\sum_{j=1}^{\infty}\beta^j\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+j} - \tilde{r}_{D,t+j}\right)\right] + \beta\frac{\overline{Y}}{\overline{A}_N}y_{t+1} + \beta E_{t+1}\left[\sum_{j=1}^{\infty}\beta^j\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+1+j} - \tilde{r}_{D,t+1+j}\right)\right]$$

$$r_{N,t+1} = -E_t\left[\sum_{j=1}^{\infty}\beta^j\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+j} - \tilde{r}_{D,t+j}\right)\right] + \beta\frac{\overline{Y}}{\overline{A}_N}y_{t+1} + E_{t+1}\left[\sum_{j=2}^{\infty}\beta^j\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+j} - \tilde{r}_{D,t+j}\right)\right]$$

$$r_{N,t+1} - \beta\tilde{r}_{D,t+1} = (E_{t+1} - E_t)\left[\sum_{j=1}^{\infty}\beta^j\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+j} - \tilde{r}_{D,t+j}\right)\right]$$

$$r_{N,t+1} = \beta\tilde{r}_{D,t+1} + (E_{t+1} - E_t)a_{N,t+1}$$

Impose AR(1), which implies $a_{N,t} = \frac{\rho\beta}{1-\rho\beta}\left(\frac{\overline{Y}}{\overline{A}_N}y_t - \tilde{r}_{D,t}\right)$ and $E_t a_{N,t+1} = \frac{\rho^2\beta}{1-\rho\beta}\left(\frac{\overline{Y}}{\overline{A}_N}y_t - \tilde{r}_{D,t}\right)$

$$r_{N,t+1} = \beta\tilde{r}_{D,t+1} + \frac{\rho\beta}{1-\rho\beta}\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+1} - \tilde{r}_{D,t+1}\right) - \frac{\rho^2\beta}{1-\rho\beta}\left(\frac{\overline{Y}}{\overline{A}_N}y_t - \tilde{r}_{D,t}\right)$$

$$r_{N,t+1} = \beta\tilde{r}_{D,t+1} + \frac{\rho\beta}{1-\rho\beta}\left(\frac{\overline{Y}}{\overline{A}_N}y_{t+1} - \tilde{r}_{D,t+1} - \rho(\frac{\overline{Y}}{\overline{A}_N}y_t - \tilde{r}_{D,t})\right)$$

(probably keep this in expectatiosn before going to AR.... thats what gourinchas and coeurdacier do... look at their language for how to describe this logic)

D.1 Log-linearizing capital returns

We define capital returns as

$$\tilde{R}_{DK,t+1} \equiv \frac{1}{Q_t}\left(\frac{R_{K,t+1}}{P_{t+1}} + Q_{t+1}(1-\delta)\right) \tag{64}$$

Log linearize to get

$$\overline{\tilde{R}}_{DK}(\tilde{r}_{DK,t+1} + q_t) = \left(\overline{\frac{R_K}{QP}}(r_{K,t+1} - p_{t+1}) + \overline{G_Q}(1-\delta)q_{t+1}\right) - \overline{\tilde{R}}_{DK}q_t \tag{65}$$

Q_t is typically nonstationary, so this log-linearization is instead written with the assumption that it has a stationary trend, with average growth factor $\overline{G_Q}$. $\overline{\frac{R_K}{QP}}$ is the average real capital income, deflated by the investment goods price. Finally, we use $\overline{\tilde{R}}_{DK} = 1/\beta$ (implied by the steady state of Equation (14)) to derive

$$\tilde{r}_{DK,t+1} = \beta\left(\overline{\frac{R_K}{QP}}(r_{K,t+1} - p_{t+1}) + \overline{G_Q}(1-\delta)q_{t+1}\right) - q_t \tag{66}$$

E Panel Vector Autoregression

Here show the VAR results (I have already calculated these)

F New Keynesian Phillips Curve

Derive New Keynesian equations here.

AR(1) coefficient	Agg Arith	Disag Arith	Agg Geom	Disag Geom	Agg Arith 5yr wts	Disag Arith 5yr wts	Agg Arith 5yr q-wts	Disag
(0,0.7]	1	33	0	27	73	59	63	
(0.7,0.8]	4	37	6	29	11	16	27	
(0.8,0.85]	15	24	6	14	20	21	12	
(0.85,0.9]	48	29	23	24	18	21	15	
(0.9,0.95]	137	81	164	107	3	6	6	
(0.95,1]	0	1	6	4	0	1	0	
Total	205	205	205	205	137	137	133	

Table 8: Distribution of autoregressive coefficients: Full sample

Advanced economies	Australia, Austria, Belgium, Canada, Switzerland, Cyprus, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland,Iceland, Israel, Italy, Japan, Korea. Rep., Malta, Netherlands, Norway, New Zealand, Portugal, Singapore, Sweden, United States
Emerging economies	Argentina, Bahrain, Brazil, Chile, China, Colombia, Costa Rica, Dominican Republic, Algeria, Ecuador, Egypt. Arab Rep., Fiji, Gabon, Guatemala, Hungary, Indonesia, India, Iran IR, Jamaica, Jordan, Kuwait, Libya, Sri Lanka, Morocco, Mexico, Mauritius, Malaysia, Pakistan, Panama, Peru, Philippines, Paraguay, Saudi Arabia, El Salvador, Syrian Arab Republic, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uruguay, Venezuela. RB, South Africa
Low income developing countries	Burundi, Burkina Faso, Bolivia, Cote d'Ivoire, Cameroon, Congo. Rep., Ethiopia, Ghana, Gambia. The,Honduras, Haiti, Kenya, Madagascar, Myanmar, Niger, Nigeria, Nicaragua, Nepal, Papua New Guinea, Rwanda, Sudan, Senegal, Sierra Leone, Togo, Zambia, Zimbabwe

Table 9: Sample countries

Table 10: Total dynamic effect for full dataset and subsamples

Sample	GDP	Cons	Cons/GDP	Investment	Exports	Imports	Agg. hrs	Cons/hr	Real ER	Nom ER	NFDA
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
All countries	1.643	3.216***	0.313	6.019***	-1.469	7.879***	0.385	5.027***	-1.418**	-0.12	4.986***
	(1.156)	(0.755)	(0.211)	(0.868)	(1.333)	(1.08)	(0.738)	(1.836)	(0.687)	(6.848)	(1.572)
Advanced Economies	3.866	4.907	0.089	6.797*	-4.384	10.236*	-0.083	4.174**	-2.284*	-11.272	2.571
	(3.273)	(4.146)	(0.588)	(4.011)	(5.737)	(5.261)	(2.117)	(1.886)	(1.382)	(7.182)	(6.01)
Emerging markets	1.43	3.736***	0.312	5.205***	-1.775	8.68***	0.563	6.631*	0	11.25	4.17*
	(1.395)	(0.83)	(0.204)	(0.888)	(2.055)	(1.758)	(0.543)	(3.568)	(0.91)	(9.47)	(2.469)
Low Income	0.504	0.64	-0.023	4.407*	-1.129	4.248**			-1.587	10.655	3.652***
	(1.359)	(1.044)	(0.381)	(2.619)	(1.664)	(1.882)			(1.542)	(16.994)	(0.831)

Note: $p < 0.1^{*}$; $p < 0.05^{**}$; $p < 0.01^{***}$
Robust standard errors reported in parentheses