Why are Countries Exposed to Nominal Exchange Rates?*

Jonathan J Adams†**and Philip Barrett**‡

WORKING PAPER This Version: October 16, 2017

Link to Most Current Version

Abstract

Most countries hold large gross asset positions, lending in their domestic currency and borrowing in foreign currency. As a result, their balance sheets are exposed to nominal exchange rate movements. We argue that when asset markets are incomplete, nominal exchange rate exposure allows countries to partially insure against shocks that move real exchange rates. We demonstrate that asset market incompleteness which features a meaningful portfolio choice can simultaneously generate realistic gross asset positions and also resolve the Backus-Smith puzzle: that relative consumptions and real exchange rates are negatively correlated. We also show that local perturbation methods that use endogenous discount factors to stabilize models are inaccurate when the average and steady state interest rates differ, even when they correctly characterize the average portfolio holdings. To address this, we develop a novel global solution method to accurately solve the equilibrium portfolio problem.

JEL-Codes: F30, F41

Keywords: Country portfolios, international business cycles, home bias, gross asset positions, exchange rates, Backus-Smith puzzle

*We are grateful for helpful comments from Thorsten Drautzburg, Burcu Eyigungor, Lars Hansen, Adrien Verdelhan, Mark Wright, and participants of the Chicago Economic Dynamics working group, the Midwest Macro Conference, the University of Florida economics seminar, the Royal Economic Society Conference, and the Federal Reserve Bank of Philadelphia. This paper previously circulated as "Resolving International Macro Puzzles with Imperfect Risk Sharing and Global Solution Methods". This paper does not represent the views of the IMF, its board, or management

†University of Florida. Website: www.jonathanjadams.com Email: adamsjonathan@ufl.edu

‡International Monetary Fund. Website: www.pobarrett.com Email: pbarrett@imf.org

1 Introduction

Do gross international asset positions matter for macroeconomic outcomes? In this paper, we argue that they do. Most countries choose gross asset positions that are long in their domestic currency and short in foreign currency: Lane and Shambaugh (2010a) document that the median country's net exchange rate exposure is 42% of GDP[1]. Until recently, modern international macroeconomics has ignored the implications of these asset positions for tractability – linearization is not well-suited for models with meaningful portfolio problems. But when balance sheet exposure is so large, macroeconomic shocks may have large wealth effects, so macroeconomic theories must account for gross asset positions to be quantitatively useful.[2] In this paper, we make three contributions to the understanding of this issue:

1. Both gross international asset positions and correlations between major macroeconomic variables are sensitive to asset market structure. So macroeconomists should consider asset markets that are consistent with data on both gross assets an key macroeconomic correlations. We demonstrate that imperfect risk-sharing with nominal noncontingent bonds is a simple asset market restriction which generates both gross asset positions with realistic exchange rate exposure and also resolves the Backus-Smith puzzle.[3]

2. We show that the default solution approach, local perturbation with ad hoc stabilizing assumptions, is a poor approximation to the true model and results in inaccurate

[1] i.e. when a country's currency depreciates by 1%, the value of its balance sheet loses 0.42% of GDP, ceteris paribus.

[2] In examining home bias in bonds, we join a large recent literature researching exchange rate exposure and valuation effects on international portfolios, including Cavallo and Tille (2006), Tille (2008), Benigno (2009), Lee, Ghironi, and Rebucci (2009), Matsumoto and Engel (2009), Mendoza, Quadrini, and RiosRull (2009), Lane and Shambaugh (2010b), Corsetti, Dedola, and Leduc (2014), and Maggiori, Neiman, and Schreger (2017) among many others. Gourinchas and Rey (2014) provide a summary of the literature on valuation effects.

[3] To the best of our knowledge, ours is the first paper which features endogenous portfolio decisions and achieves a realistic consumption-real exchange rate correlation. Coeurdacier and Rey (2013) note that most models with endogenous portfolios recover perfect risk-sharing, and that in papers where asset markets are incomplete, consumption and real exchange rates typically remain correlated, as in Benigno and Kucuk-Tuger (2008) and Coeurdacier, Kollmann, and Martin (2010).

predictions for the Backus-Smith correlation and other moments.

3. Because local perturbation methods fail to recover the true equilibrium dynamics, we develop a global solution algorithm generalizing Maliar and Maliar (2015)'s projections method, and demonstrate its accuracy. The accuracy gain of this method over a local one arise because it approximates near the average, not the steady state, interest rate.

The Backus-Smith puzzle is a well-known indicator that macroeconomic dynamics depend on asset markets, and we show that it is easily resolved when countries face a realistic and nontrivial portfolio choice. Backus and Smith (1993) show that when asset markets are complete, models of international business cycles generate strong positive correlations between a country's consumption and their real exchange rate. In contrast, empirical estimates of this correlation vary across countries but are typically negative. The source of the correlation in a complete markets model is the equilibrium condition equating the real exchange rate to the ratio of marginal utilities. The most obvious way to resolve the consumption-real exchange rate puzzle is to break this link by introducing asset market incompleteness. However, early attempts to pursue this directly failed to yield much quantitative success (see, for example Chari, Kehoe, and McGrattan (2002)). And so the subsequent literature turned to resolving the puzzle with additional model ingredients or frictions. We demonstrate that the intuition of these early attempts to explain the puzzle was valid, but over-simplified the asset market, eliminating a crucial mechanism: portfolio choice.

In this paper, we revisit the issue and argue that asset market incompleteness can explain the puzzle, but the type of asset incompleteness matters. We show that when households are allowed to trade nominal bonds, asset market restrictions *can* produce negative correlations of consumption and the real exchange rate. The key mechanism that produces this result is households' use of asymmetric holdings of foreign and domestic nominal bonds to hedge their income risk. Prior attempts to resolve the puzzle missed this mechanism by examining asset markets without all of these ingredients. First, some papers considered only real bonds, to no avail: Lustig and Verdelhan (2016) prove that any portfolio decision over real non-contingent bonds yields a lower bound of zero for the Backus-Smith correlation. Second, many papers reduced the set of assets to a single nominal bond, so that standard perturbation methods could be employed without having to solve a portfolio decision over risky assets, as in Chari, Kehoe, and McGrattan (2002), thus eliminating any role for gross asset positions. In contrast, we allow households to choose their holdings of multiple nominal bonds. As a result, the equilibrium conditions of the portfolio decision over nominal bonds in this model can resolve the Backus-Smith puzzle.

We present a two-country model driven by two types of shocks - real shocks to productivity and nominal shocks to prices of country-specific tradable intermediates. We consider a simple model with only the ingredients necessary to both produce home bias in bonds[4] and resolve Backus-Smith. The crucial asset market restriction we impose is that households may only hold non-contingent nominal bonds. Our structure implies that real shocks generate a positive correlation between labor income and foreign asset returns; higher domestic productivity makes the foreign intermediate relatively scarcer, raising both the real and nominal exchange rates. Households use this correlation to hedge labor income risk, taking an asymmetric asset position reflecting that seen in the data - going relatively long domestic bonds. However, because this hedge comes at the cost of increasing households' exposure to nominal shocks, they do not go long enough to fully insure the real shocks. This portfolio loses value when the nominal exchange rate rises, which can reduce consumption if the portfolio is sufficiently home biased. So if nominal and real exchange rates covary, the Backus-Smith correlation can be negative.

[4] Home bias in bond holdings is a also component of the broader home bias in assets, which is puzzling in standard international RBC models; see Lewis (1999) for an early summary, and Coeurdacier and Rey (2013) for a more recent one. When asset returns are uncorrelated with labor income, as in Lucas Jr. (1982), households fully diversify. When domestic asset returns are positively correlated with labor income, households bias their portfolios towards foreign assets, as in Baxter and Jermann (1997). In our model, domestic bond returns are negatively correlated with labor income, so households bias their portfolios towards domestic bonds.

The Backus-Smith correlation is sensitive to parameterization of endogenous discount factors (EDFs). In open economy models, approximations around a deterministic steady state are typically nonstationary, so economists assume ad hoc features to induce stationarity, most commonly an EDF which is decreasing in the consumption level (Schmitt-Grohe and Uribe (2003)). But in general equilibrium, standard international macroeconomic models are typically stationary[5], so the ad hoc endogenous discount factor assumption is unnecessary when using global solution methods. Recently, Rabitsch, Stepanchuk, and Tsyrennikov (2015) have shown that policy functions can be inaccurate when an EDF is assumed, and we confirm that an EDF changes the dynamics in our model dramatically. The Backus-Smith correlation is particularly sensitive to the parameterization of the EDF, even when the EDF's elasticity is small. This is troubling for macroeconomic modelers, because the EDF is a commonly used ad hoc assumption which is not well disciplined by the data.

To achieve accurate global solutions, we generalize the simulation and projection algorithm of Maliar and Maliar (2015) to a model where equilibrium conditions cannot easily be expressed purely in terms of the states.[6] As such, our paper is proof-of-concept that highly accurate global solution techniques can be applied to model equation listings that include prices. We use global methods because our model contains no ad hoc endogenous discount factor chosen to impose stationarity in the linear model. We show that our global method is substantially more accurate than the standard perturbation method for international models with portfolio problems, the Devereux-Sutherland algorithm.[7] The critical advantage of our global method is that we can solve near the true average interest rate, which is less than in the deterministic steady state.

[5] Stationarity holds because on average, interest rates are less than the discount rate.

[6] This algorithm joins the large literature using projection methods to solve macroeconomic models, which Fernndez-Villaverde, Rubio-Ramrez, and Schorfheide (2016) survey.

[7] The Devereux-Sutherlan algorithm is presented in Devereux and Sutherland (2011), was developed independently and concurrently by Tille and van Wincoop (2010), and is closely related to the method of Evans and Hnatkovska (2012). Papers that use Devereux-Sutherland to solve endogenous portfolio problems with nominal bonds include Rahbari (2009), Berriel and Bhattarai (2013), Coeurdacier and Gourinchas (2016)

2 Model

We consider a two-country RBC model, similar to the workhorse model of Backus, Kehoe, and Kydland (1992) but with two goods and without capital. There are two identical countries, indexed by $i = H$ and $i = F$, which we refer to as Home and Foreign. They are each populated by mass one of identical, infinitely lived households. Each country produces a tradable intermediate good and a nontradable consumption good. Consumption in each country aggregates the nontradable consumption good with a tradable consumption bundle, which itself is an aggregate of domestic and foreign intermediates.

2.1 Households

The representative household in country i maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} \tag{1}$$

where $C_{i,t}$ is the domestic consumption good for country i in period t.

The household earns wage income $W_{i,t}$ per unit of labor, denominated in domestic currency[8]. The household inelastically supplies 1 unit of labor every period. The price level of the consumption good is $P_{i,t}$. The household has access to two asset markets. It can hold non-contingent domestic bonds $B^i_{i,t+1}$ at price $\frac{1}{R^i_{t+1}}$, which pays one unit of domestic currency in period $t+1$, and it can hold non-contingent foreign bonds $B^j_{i,t+1}$ at price $\frac{1}{R^j_{t+1}}$, which pays one unit of foreign currency. Both of these bonds are in zero net supply. $\mathcal{E}^{i,j}_t$ is the nominal exchange rate: the relative price of the country j consumption good in country j's currency, to the price of the country i consumption good in country i's currency.

[8] "Currency" here serves the role only of a unit of account, not a means of exchange nor a store of value. Given that we are most interested in wealth effects induced by relative fluctuations in competing units of account, this is an appropriate simplification for our purposes.

The household's period budget constraint (denominated in domestic currency) is

$$W_{i,t} + B^i_{i,t} + \mathcal{E}^{i,j}_t B^j_{i,t} = P_{i,t}C_{i,t} + \frac{B^i_{i,t+1}}{R^i_{t+1}} + \mathcal{E}^{i,j}_t \frac{B^j_{i,t+1}}{R^j_{t+1}} \tag{2}$$

for $j \neq i$.

2.2 Firms

There are four types of firms in each country: intermediate goods producers, nontradable goods producers, tradable consumption goods producers, and final goods producers. All firms are perfectly competitive.

The intermediate goods producers hire domestic labor $N^T_{i,t}$ to produce a country specific tradable intermediate good $X_{i,t}$. Their production function is linear in labor, with productivity $A^T_{i,t}$, which is exogenous and stochastic

$$X_{i,t} = A^T_{i,t} N^T_{i,t} \tag{3}$$

And $A^T_{i,t}$ follows the process:

$$\log A^T_{i,t} = \rho \log A^T_{i,t-1} + \epsilon^T_{i,t} \qquad\qquad \epsilon^T_t \sim_{i.i.d.} N(0, \sigma^T)$$

The nontradable goods producers hire domestic labor $N^N_{i,t}$ to produce a country specific nontradable good $C^N_{i,t}$. Their production function is linear in labor, with productivity $A^N_{i,t}$:

$$C^N_{i,t} = A^N_{i,t} N^N_{i,t} \tag{4}$$

$A^N_{i,t}$ is constant, so we normalize it to 1.

Foreign and domestic tradable goods aggregate into a tradable consumption good $C^T_{i,t}$ by

$$C^T_{i,t} = (\alpha^{\frac{1}{\eta}} (X^i_{i,t})^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} (X^j_{i,t})^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} \tag{5}$$

The final goods producers in each country produce the consumption good from the tradable consumption good $C^T_{i,t}$ and the nontradable good $C^N_{i,t}$. Their production function is CES with elasticity of substitution ξ and tradable expenditure share μ

$$C_{i,t} = (\mu^{\frac{1}{\xi}}(C_{i,t}^T)^{\frac{\xi-1}{\xi}} + (1-\mu)^{\frac{1}{\xi}}(C_{i,t}^N)^{\frac{\xi-1}{\xi}})^{\frac{\xi}{\xi-1}} \tag{6}$$

The price of intermediate good $X_{i,t}^j$ in units of the country i consumption good is $P_{i,t}^j$, so intermediate goods firms solve

$$\max_{N_{i,t}^T} P_{i,t}^i A_{i,t}^T N_{i,t}^T - W_{i,t} N_{i,t}^T \tag{7}$$

The price of the nontradable good $C_{i,t}^N$ in units of the country i consumption good is $P_{i,t}^N$, so nontradable goods firms solve

$$\max_{N_{i,t}^N} P_{i,t}^N A_{i,t}^N N_{i,t}^N - W_{i,t} N_{i,t}^N \tag{8}$$

Tradable goods firms aggregate the domestic and foreign intermediate goods. Where $P_{i,t}^T$ is the price of the tradable consumption good, the tradable consumption goods firms solve

$$\max_{C_{i,t}^T, C_{i,t}^N} P_{i,t}^T(\alpha^{\frac{1}{\eta}}(X_{i,t}^i)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}}(X_{i,t}^j)^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}} - P_{i,t}^i X_{i,t}^i - P_{i,t}^j X_{i,t}^j \tag{9}$$

Final goods firms in country i face prices $P_{i,t}^T$ for the tradable consumption good and $P_{i,t}^N$ for the nontradable good. They solve

$$\max_{X_{i,t}, X_j,t} P_{i,t}(\mu^{\frac{1}{\xi}}(C_{i,t}^T)^{\frac{\xi-1}{\xi}} + (1-\mu)^{\frac{1}{\xi}}(C_{i,t}^N)^{\frac{\xi-1}{\xi}})^{\frac{\xi}{\xi-1}} - P_{i,t}^T C_{i,t}^T - P_{i,t}^N C_{i,t}^N \tag{10}$$

2.3 Goods Prices

The first order conditions for both the intermediate goods firms and the nontradable goods firms imply that wages are proportional to their respective value marginal products of labor:

$$P_{i,t}^i A_{i,t}^T = W_{i,t} \tag{11}$$

$$P_{i,t}^N A_{i,t}^N = W_{i,t} \tag{12}$$

The first order conditions for the tradable consumption goods firms imply

$$(\alpha\frac{C^T_{i,t}}{X^i_{i,t}})^{\frac{1}{\eta}} = \frac{P^i_{i,t}}{P^T_{i,t}} \qquad ((1-\alpha)\frac{C^T_{i,t}}{X^j_{i,t}})^{\frac{1}{\eta}} = \frac{P^j_{i,t}}{P^T_{i,t}} \qquad [j \neq i] \tag{13}$$

and the first order conditions for the final goods firms imply

$$(\mu\frac{C_{i,t}}{C^T_{i,t}})^{\frac{1}{\xi}} = \frac{P^T_{i,t}}{P_{i,t}} \qquad ((1-\mu)\frac{C_{i,t}}{C^N_{i,t}})^{\frac{1}{\xi}} = \frac{P^N_{i,t}}{P_{i,t}} \qquad [j \neq i] \tag{14}$$

The real exchange rate is the price of foreign relative to domestic consumption. The law of one price holds for both intermediate goods, so the nominal exchange rate $\mathcal{E}^{i,j}_t$ is

$$\mathcal{E}^{i,j}_t = \frac{P^i_{i,t}}{P^i_{j,t}} = \frac{P^j_{i,t}}{P^j_{j,t}} \tag{15}$$

and the real exchange rate $Q^{i,j}_t$ as

$$Q^{i,j}_t = \frac{P_{j,t}}{P_{i,t}}\mathcal{E}^{i,j}_t \tag{16}$$

The terms of trade in this economy is the relative price of the foreign intermediate in terms of the domestic intermediate:

$$S^{i,j}_t = \frac{P^j_{i,t}}{P^i_{i,t}} = \frac{P^j_{j,t}}{P^i_{j,t}} = \frac{1}{S^{j,i}_t} \tag{17}$$

2.4 The Price Level

Without further discipline, the price level is undetermined. So for both countries, assume that the domestic intermediate price is exogenous and stochastic, and follows the process:

$$\log P^i_{i,t} = \rho \log P^i_{i,t-1} + \epsilon^P_{i,t} \qquad\qquad \epsilon^P_{i,t} \sim_{i.i.d.} N(0, \sigma^P) \tag{18}$$

We assume that the price level is stochastic so that perfect risk-sharing is impossible. If the only shocks were the two productivity shocks, households could choose portfolios of the two assets to recover perfect risk-sharing, as in Devereux and Sutherland (2007) and Engel and Matsumoto (2009). With four shocks, asset markets are incomplete, so relative consumption and the real exchange rate will not be perfectly correlated. Because two of the shocks are nominal, it is crucial that bonds are also nominal; if households were trading real bonds instead, perfect risk-sharing would be attainable.

One interpretation of this assumption is that the domestic monetary authorities in both countries commit to stabilizing domestic tradable producer prices, but do so with error. Alternatively, this assumption could be thought of as implying that domestic tradable producer prices are very sticky, but that this price stickiness does not have real effects on the production side of the economy as it does in Chari, Kehoe, and McGrattan (2002). Rather, it determines how inflation behaves in response to real shocks[9]. And inflation dynamics will have real effects through the household balance sheet.

The price of the aggregate consumption good is the CES price index

$$P_{i,t} = (\mu(P^T_{i,t})^{1-\xi} + (1-\mu)(P^N_{i,t})^{1-\xi})^{\frac{1}{1-\xi}} \tag{19}$$

and the price of the tradable consumption good $C^T_{i,t}$ is the CES price index

$$P^T_{i,t} = (\alpha(P^i_{i,t})^{1-\eta} + (1-\alpha)(P^j_{i,t})^{1-\eta})^{\frac{1}{1-\eta}} \tag{20}$$

When $P^i_{i,t}$ is fixed at 1, the terms of trade $S^{i,j}_t$ moves one for one with the nominal exchange rate $\mathcal{E}^{i,j}$. An increase in domestic productivity $A_{i,t}$ relative to foreign productivity will increase the terms of trade, increasing the nominal exchange rate and the price level, which is why productivity growth is positively correlated with inflation in equilibrium.

2.5 Asset Prices

The household's optimal behavior includes an Euler Equation for each asset. The domestic bond's Euler equation is

$$1 = R^i_{t+1}\beta E_t\left[\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\gamma}\frac{P_{i,t}}{P_{i,t+1}}\right] \tag{21}$$

[9] Indeed, in the calibrated solution of Section 3, we allow for a richer shock structure, with within-country correlation of the real and nominal shocks.

while the Euler equation for the foreign bond $j \neq i$ is

$$1 = R^j_{t+1} \beta E_t \left[\frac{\mathcal{E}^{i,j}_{t+1}}{\mathcal{E}^{i,j}_t} \left(\frac{C_{i,t}}{C_{i,t+1}} \left(\right)^{\gamma} \frac{P_{i,t}}{P_{i,t+1}} \right] \tag{22}$$

Both bonds suffer inflation risk due to variation in the price of consumption, while the foreign bond also suffers exchange rate risk because $\frac{\mathcal{E}^{i,j}_{t+1}}{\mathcal{E}^{i,j}_t}$ varies stochastically in equilibrium.

Given the same interest rate, a household has incentive to decrease its holdings of foreign bonds if exchange rate growth (currency depreciation) $\log \frac{\mathcal{E}^{i,j}_{t+1}}{\mathcal{E}^{i,j}_t} \equiv e^{i,j}_{t+1}$ covaries sufficiently with consumption growth $\log \frac{C_{i,t+1}}{C_{i,t}} \equiv c_{i,t+1}$. This is central force that leads to home bias in assets; households choose to be long domestic bonds and short foreign bonds to minimize the correlation of their wage income with the returns on foreign bonds.

A second order approximation of the Euler equations provides a condition for home bias in assets. The second order log approximations[10] around the period $t+1$ average for the domestic Euler equation[11] (with lower case variables denoting growth rates) is

$$0 \approx \log R^i_{t+1} + \log \beta - \gamma E_t c_{i,t+1} - E_t \pi_{i,t+1} + \gamma^2 \frac{Var_t(c_{i,t+1})}{2} + \frac{Var_t(\pi_{i,t+1})}{2} + \gamma Cov_t(c_{i,t+1}, \pi_{i,t+1}) \tag{23}$$

where inflation is defined $\log \frac{P_{i,t+1}}{P_{i,t}} \equiv \pi_{i,t+1}$. The Euler equation for the foreign bond $j \neq i$ is

$$\begin{aligned} 0 \approx \log R^j_{t+1} + \log \beta + E_t e^{i,j}_{t+1} - \gamma E_t c_{i,t+1} - E_t \pi_{i,t+1} + \frac{Var_t(e^{i,j}_{t+1})}{2} + \gamma^2 \frac{Var_t(c_{i,t+1})}{2} + \frac{Var_t(\pi_{t+1})}{2} \\ - \gamma Cov_t(e^{i,j}_{t+1}, c_{i,t+1}) - Cov_t(e^{i,j}_{t+1}, \pi_{i,t+1}) + \gamma Cov_t(c_{t+1}, \pi_{i,t+1}) \end{aligned} \tag{24}$$

Differencing the two Euler equations yields an approximate indifference condition for holding the two bonds:

$$\log R^j_{t+1} + E_t e^{i,j}_{t+1} - \log R^i_{t+1} \approx \gamma Cov_t(e^{i,j}_{t+1}, c_{i,t+1}) + Cov_t(e^{i,j}_{t+1}, \pi_{i,t+1}) - \frac{Var_t(e^{i,j}_{t+1})}{2} \tag{25}$$

[10] All approximations in this section are derived in appendix A.

[11] Consistent with our critique of linearization solutions, this argument is not dependent on finding a steady state. It is an approximation around the conditional expectation in the next period give the current state.

The left hand side of this approximation is the premium of foreign bonds over domestic bonds, which is zero when uncovered interest rate parity holds. We use this equation to explain the intuition behind our main results.

2.6 Home Bias and the Backus-Smith Puzzle

How do households choose their portfolio allocations? When optimizing, they must satisfy approximation (25). The symmetry of the model implies that uncovered interest rate parity holds on average. Taking unconditional expectations of this approximation therefore yields:

$$0 \approx \gamma Cov(e_{t+1}^{i,j}, c_{i,t+1}) + Cov(e_{t+1}^{i,j}, \pi_{i,t+1}) - \frac{Var(e_{t+1}^{i,j})}{2} \tag{26}$$

The most obvious implication of this approximation is that when risk aversion γ is large, the covariance of consumption and the nominal exchange rate must be small in absolute value. The incomplete markets framework can therefore produce a weak correlation between at least the nominal exchange rate and consumption.

In addition to simply being small, depending on the other terms in this approximation, the consumption-nominal exchange rate correlation may also be negative. In particular, if $Cov(e_{t+1}^{i,j}, \pi_{i,t+1}) - \frac{Var(e_{t+1}^{i,j})}{2} > 0$ (a result which holds in some calibrations of the model) then this approximation dictates that $\gamma Cov(e_{t+1}^{i,j}, c_{i,t+1}) < 0$.

The mechanism whereby this occurs is through households' adjustment of their international bond portfolio. If households were to hold foreign and domestic bonds equally, we would expect $Cov(e_{t+1}^{i,j}, c_{i,t+1}) > 0$, because their total income would covary positively with the nominal exchange rate. In order to deliver $\gamma Cov(e_{t+1}^{i,j}, c_{i,t+1}) < 0$, they go long domestic bonds and short foreign bonds in order to reduce their exposure to exchange rate risk. Therefore, home bias in bond holdings is crucial part of explaining the consumption-real exchange rate anomaly; households achieve a low (or even negative) correlation of the consumption with the real exchange rate by shorting foreign assets.

The Backus-Smith puzzle concerns consumption and real exchange rates, but Equation (26) only characterizes the relationship between consumption and nominal exchange rates. To understand the covariance of relative consumption with the real exchange rate, express approximation (25) in real terms, by defining real exchange rate growth as $\log \frac{Q_{t+1}^{i,j}}{Q_t^{i,j}} \equiv q_{t+1}^{i,j}$. Then differencing this condition for country H and country F implies

$$0 \approx Cov_t(e_{t+1}^{i,j}, c_{i,t+1} - c_{j,t+1} - \frac{1}{\gamma} q_{t+1}^{i,j}) \tag{27}$$

The implication of this approximation is that deviations from the complete markets consumption ratio are uncorrelated with exchange rate growth, which is the risky premium of foreign over domestic bonds[12]. Then use the relationship $e_{t+1}^{i,j} = q_{t+1}^{i,j} + \pi_{i,t+1} - \pi_{j,t+1}$ to rearrange this approximation in terms of the covariance between relative consumption and the real exchange rate:

$$Cov_t(c_{i,t+1} - c_{j,t+1}, q_{t+1}^{i,j}) \approx \frac{1}{\gamma} Cov_t(q_{t+1}^{i,j}, e_{t+1}^{i,j}) - Cov_t(c_{i,t+1} - c_{j,t+1}, \pi_{i,t+1} - \pi_{j,t+1}) \tag{28}$$

This approximation reveals why the portfolio decision can resolve the real exchange rate consumption puzzle. $Cov_t(q_{t+1}^{i,j}, e_{t+1}^{i,j})$ is positive, so if the intertemporal elasticity of substitution $\frac{1}{\gamma}$ is small, then the covariance of the real exchange rate and relative consumption will be negative if consumption covaries more with domestic inflation than foreign inflation. This holds in our model because productivity shocks make households richer, and so consume more, but at the same time cause a depreciation of the currency and so increase imported inflation.

2.7 Equilibrium

A competitive equilibrium in this economy consists of sequences for $t \geq 0$ of prices, $P_{H,t}$, $P_{F,t}$, $P_{H,t}^H$, $P_{F,t}^H$, $P_{H,t}^F$, $P_{F,t}^F$, $W_{H,t}$, $W_{F,t}$, $\mathcal{E}_t^{H,F}$, R_t^H, R_t^F; allocations, $C_{H,t}$, $C_{F,t}$, $N_{H,t}$, $N_{F,t}$, $X_{H,t}^H$, $X_{F,t}^H$,

[12] Looking ahead to the solution techniques we discuss later, we note that equation (27) is *precisely* that used by Devereux and Sutherland (2011) to generate linearized solutions to the portfolio problem.

$X^F_{H,t}$, $X^F_{F,t}$; and assets $B^H_{H,t}$, $B^H_{F,t}$, $B^F_{H,t}$, $B^F_{F,t}$; given initial assets $B^H_{H,0}$, $B^H_{F,0}$, $B^F_{H,0}$, $B^F_{F,0}$ and realizations of the exogenous stochastic productivities $A_{H,t}, A_{F,t}$; such that:

1. Households maximize their intertemporal utility (1).

2. Firms maximize profits, satisfying factor demands (11), (12), (13) and (14).

3. Markets clear, satisfying the household budget constraint (2), the labor market constraint $N^T_{i,t} + N^N_{i,t} = 1$ and production functions (3), (4), (5) and (6).

4. Domestic intermediate prices determine the price level by satisfying (18).

5. Household assets are in net zero supply: $B^H_{H,t} + B^F_{H,t} = 0$ and $B^H_{F,t} + B^F_{F,t} = 0$

3 Results from a Calibrated Economy

In this section we discuss two main issues. First, we show that a calibration of the model including within-country correlated shocks can produce a small, negative correlation between the real exchange rate and relative consumption. The same calibration, though, fails to reproduce this fact if asset market restrictions are either tighter (the one bond case) or looser (complete markets).

The second issue we discuss is solution technique. The model is challenging to solve because a) the linearized model is non-stationary near the steady state, and b) the steady state is far from the solution manifold. We show that standard modifications that stationarize the model near the steady state produce very inaccurate peturbation solutions simply because the true solution lies far from the steady state[13]. The results of Schmitt-Grohe and Uribe (2003), who argue that this approach works well in a small open economy model, do therefore not carry over to the two-country case.

[13] This holds even for methods which produce accurate levels of asset holdings, such as Tille and van Wincoop (2010) and Devereux and Sutherland (2011), because they too rely on linearizing near the steady state interest rate.

3.1 Calibrated solution

To illustrate how the model works to deliver a negative correlation between the real exchange rate and relative consumption, we calibrate at a quarterly frequency using standard parameters from the literature. The main restriction we impose at this point is that the shock processes in the two countries are symmetric and independent[14]. We assume that the stochastic variables, productivity and prices, follow a joint autoregressive process:

$$\begin{pmatrix} \log A_{i,t}^T \\ \log P_{i,t}^i \end{pmatrix} = B \begin{pmatrix} \log A_{i,t-1}^T \\ \log P_{i,t-1}^i \end{pmatrix} + \epsilon_{i,t} \qquad var(\epsilon_{i,t}) = \Sigma$$

To calibrate this process, we estimate a VAR for detrended log tradable productivity $\log A_{i,t}^T$ and detrended log tradable prices $\log P_{i,t}^i$ on US KLEMS data[15]. The US KLEMS data on productivity and tradable prices are annual, but we are interested in macroeconomic statistics typically measured at a quarterly frequency, such as the Backus-Smith correlation. To generate a quarterly shock process, we estimate the VAR using the annual data, and then impute a quarterly VAR consistent with the annual process[16]. The estimated persistence and innovation covariance matrices are reported in Table 1 with standard errors from Maximum Likelihood Estimation.

$$\text{Where:} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \tag{29}$$

[14] Our model is estimated as if two copies of the United States traded with one another. We select this symmetric case for three reasons. First, we choose not to estimate a pair of similar economies (e.g. the US and the Euro-zone) because we want to match the aggregate trade shares, and no two large economies have bilateral flows that are nearly as large as their total trade flows. Second, we choose not to estimate the US versus the rest of the world, because tradable/nontradable shares and prices are not available for the world as a whole. Finally, in this stylized model, we to be clear that our results are generated by the few economic ingredients, rather than an asymmetry across countries.

[15] We follow Stockman and Tesar (1995) and define tradable sectors as: agriculture, mining, manufacturing, and transportation.

[16] This interpolation is, of course, not unique. But it is if we insist that the quarterly autocorrelations of produtivity and prices are, like their annual counterparts, positive.

	b_{11}	b_{21}	b_{11}	b_{22}	σ_{11}	σ_{12}	σ_{22}
Point estimate	0.95	0.01	0.00	0.97	0.00017	-0.00009	0.00015
Standard error	0.07	0.06	0.04	0.04	0.00002	0.00001	0.00002

Table 1: VAR estimates of the shock processes for output and prices

Although cross-persistence is minimal (the off-diagonal elements of B are small) the cross-correlation of the innovations is large, at -0.6. The key model mechanism is that households use their portfolio choice to insure against competing nominal and real shocks. So including correlation of these shocks is an important test of the quantitative validity of our results[17].

The remaining parameter values are summarized in Table 2:

Parameter	Interpretation	Value	Justification
β	Discount factor	0.99	4% Annual real rate
γ	CRRA risk aversion	2	Standard value
α	Share of domestic goods in tradables	0.62	US KLEMS
η	Domestic-foreign elasticity of substitution	2	Standard value
μ	Share of tradables in consumption	0.79	US KLEMS
ξ	Tradable-nontradable elasticity of substitution	0.44	Stockman and Tesar (1995)

Table 2: Illustrative calibration

While there is not strong agreement in the literature over the true value of some of these parameters (particularly γ and η), we endeavor to choose values that are as uncontroversial as possible. We consider the sensitivity of our results to these assumptions in Section 4.

The main object we wish to measure is the correlation of relative consumption growth with the real exchange rate. That is, ρ_{QC} given by:

$$\rho_{QC} = corr\left(\log\left(Q_{t+1}/Q_t\right), \log\left(C_{1,t+1}/C_{1,t}\right) - \log\left(C_{2,t+1}/C_{2,t}\right)\right)$$

Our main contention is that risky gross asset positions matter for this correlation. So we also compare our results those of two other models, representing alternative views. The first,

[17] As suggested by Maliar and Maliar (2015), we use monomial integration to allow for rapid solution of the Euler equations with four correlated shocks.

complete markets, represents the notion that asset-based risk is not a meaningful driver of this correlation. The second, a symmetric one-bond model, captures the idea that only risks to the *net* asset position matter. In this case, we permit only an average bond, which costs one unit of each currency in period t and returns R_{t+1} units of each currency in period $t+1$. The corresponding Euler equations are then:

$$1 = R_{t+1}\beta E_t \left[\frac{1+\mathcal{E}_{t+1}^{i,j}}{1+\mathcal{E}_t^{i,j}} (\frac{C_{i,t}}{C_{i,t+1}})^{\gamma} \frac{P_{i,t}}{P_{i,t+1}} \right]$$

The calibrated model delivers a value of ρ_{QC} of -0.10. This is shown in the second line of Table 3. This value is in line with what is commonly observed in the data. Benigno and Thoenissen (2008) report a median estimate of -0.16 for advanced economies. In contrast, the complete markets model delivers a correlation of 1: the Backus-Smith puzzle. And the symmetric one-bond example produces a correlation of -1.

Model	ρ_{QC}
Complete markets, no nontradables	1
Two bonds, nontradables	-0.10
Two bonds, nontradables, EDF	[-1.0, 0.4]
One bond, nontradables	-1
Data: Beningo & Thoenissen (median)	-0.16

Table 3: Model values for ρ_{QC}

What explains these different values? When markets are complete, a correlation of 1 is an equilibrium condition, because marginal utilities must be proportional to the real exchange rate. When there is one bond and nontradables, positive productivity shocks to the tradable sector lead to consumption growth, because the risk is uninsured and makes countries richer. But tradables and nontradables are complements so the real exchange rate falls, and the Backus-Smith correlation is negative.[18]

[18] This is the same effect as studied in Benigno and Thoenissen (2008), and similar to Corsetti, Dedola, and

Yet when there are two bonds, countries are *partially* insured. After a domestic product productivity increase, the nominal exchange rate rises because the foreign tradable becomes dear relative to the domestic tradable, and their nominal prices are fixed. With two bonds, households choose a portfolio exposed to this exchange rate risk, so that their portfolio loses value after a productivity shock. Therefore the correlation between consumption growth and productivity (and thus the real exchange rate) is near zero. Home bias in the calibrated model is illustrated in Figure 1, which shows the asset distributions in a ten-million-period simulation. Saving in the domestic currency is typically funded by borrowing in the foreign currency. The slight asymmetry in the distribution results from the fact that even though assets are not a random walk, they are very persistent, so even a very long simulation results in a slightly asymmetric asset distribution. As discussed in the next section, the persistence of assets is determined by the size of the average interest rate relative to $1/\beta$. Accurately calculating the interest rate is therefore critical for describing the dynamics of the model, including the Backus-Smith correlation.

3.2 Solution Technique

Although the model itself is stationary, linearizing around the steady state interest rate will produce an average interest rate of $1/\beta$, and so deliver non-stationary debt dynamics. One approach to dealing with this issue is to modify the model to stationarize it near the steady state, as in (Schmitt-Grohe and Uribe, 2003). In following section we show that such modifications have non-trivial effects in our model. So instead, we solve the model using a nonlinear

Leduc (2008), who instead suppose that home and foreign goods are complements. Mukhin, Itskhoki, and others (2016) criticize these mechanisms as being inconsistent with the Meese-Rogoff puzzle and the Purchasing Power Parity puzzle. This criticism at least partially applies to our model; nominal exchange rates are not a random walk, but are highly correlated with real exchange rates (Figure 7). Engel (1999) also criticizes this channel, finding that tradable prices account for most of the changes in relative price levels at short horizons. Alternative mechanisms through which productivity shocks can produce consumption growth and a real exchange rate decline for some asset market structures include nonseparable utility with news shocks (Colacito and Croce, 2013), nonseparable utility with labor wedge shocks (Karabarbounis, 2014), and financial shocks Mukhin, Itskhoki, and others (2016).

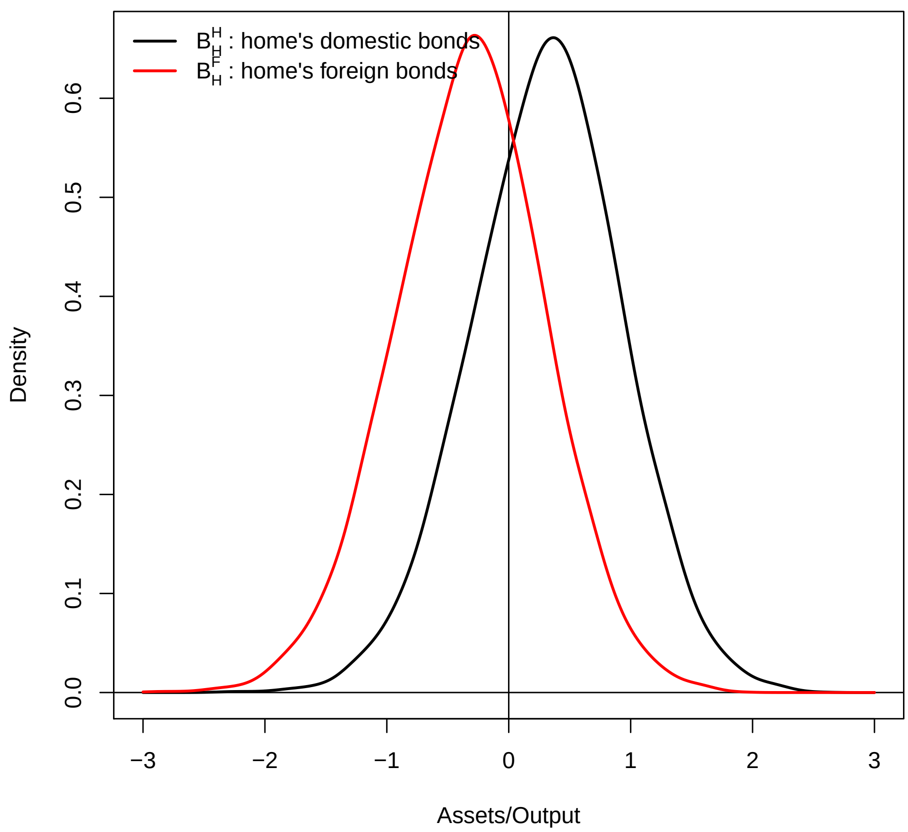

Figure 1: Distribution of bond holdings

global method, based on that of Maliar and Maliar (2015). Details of the solution method are presented in Appendix B.

3.2.1 Solving the Model near the Steady State

In our model, the deterministic steady state interest rate is $R_t^H = R_t^F = 1/\beta$. Approximating the model near this point results in explosive asset dynamics, as the average rate of interest equals the rate of time preference, and so induces a unit root in asset dynamics[19]. This is a

[19] A related issue is that of perfect substitutability of assets at the steady state. But this is a solved problem - Devereux and Sutherland (2011) show how to compute the correct approximation to asset holdings in such a model - and distinct from the one we address here. However, that technique also relies on approximating near a steady state interest rate of $1/\beta$.

common problem in small open economy models where the world interest rate is fixed, and is typically set to $1/\beta$. Schmitt-Grohe and Uribe (2003) show that a number of simple model modifications which stationarize the canonical small open economy model have no impact on a large range of economically relevant model moments. This is a key contribution to the solution of open economy models, as it allows them to be modified slightly and then solved by standard perturbation methods.

The modification that is most commonly used is to alter the consumer's preferences so that the discount rate is a function of current consumption[20]. This is often termed an "endogenous discount factor". This introduces a new parameter θ in the Euler equations, which become:

$$
\begin{aligned}
1 &= R^{i}_{t+1}\beta C^{\theta}_{i,t} E_t \left[\left(\frac{C_{i,t}}{C_{i,t+1}} \right)^{\gamma} \frac{P_{i,t}}{P_{i,t+1}} \right] \\
1 &= R^{j}_{t+1}\beta C^{\theta}_{i,t} E_t \left[\frac{\mathcal{E}^{i,j}_{t+1}}{\mathcal{E}^{i,j}_{t}} \left(\frac{C_{i,t}}{C_{i,t+1}} \right( )^{\gamma} \frac{P_{i,t}}{P_{i,t+1}} \right]
\end{aligned}
$$

The idea behind this approach is to pick a θ large enough that a local approximation is stationary, but small enough that the underlying economic properties of the model are unaffected. Figure 2 shows that this is not possible in the current model. The left hand panel displays the Backus-Smith coefficient as a function of θ, as well as that resulting from our global solution method[21]. Small changes in θ have a very large effect on this coefficient. As θ varies from 0 to 0.02, the Backus-Smith statistic ranges from nearly -1.0 to nearly 0.5. The true value of the Backus-Smith statistic, computed via the global solution, is shown by the horizontal dashed line. In other words, the choice of the ad hoc problem modification is economically meaningful.

The right hand panel of Figure 2, which shows the standard deviation of domestic assets

[20] This is also the only one of Schmitt-Grohe and Uribe's modifications that doesn't alter the exact mechanism that we are trying to investigate: their ad hoc debt elastic demand curve is an endogenous feature of our model; portfolio adjustment costs would alter the portfolio decision that we want to study; and complete markets are precisely the paradigm that we know fails to produce empirically plausible Backus-Smith correlations.

[21] When we solve the model using a local linearization, we make sure to approximate around the true asset level using the approach of (Devereux and Sutherland, 2011).

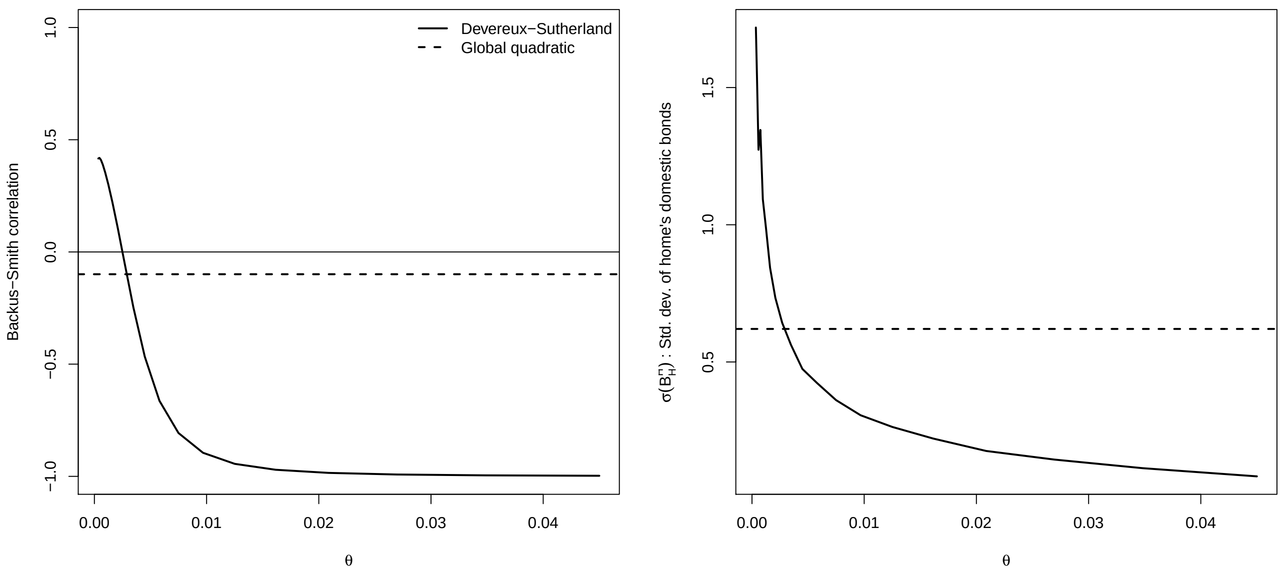

Figure 2: Properties of the local linear solution

as a function of θ, which explains why this sensitivity occurs. Reducing θ brings the local model dynamics closer to a unit root, causing the standard deviation of assets to increase without bound. So although reducing θ brings the modified model closer to the true one, it also reintroduces the very problem that the endogenous discount factor was designed to solve.

The main lesson from Figure 2 is that Schmitt-Grohe and Uribe's main result for small open economies - that closing the model with an EDF has no substantial economic effect - does not carry over to a two-country model[22]. The equilibrium dynamics are not robust to the parameterization of the stabilizing assumption. The reason that the approach fails here is that the endogenous discount factor is designed to solve a problem that does not exist when the international asset market clears. In small open economies, the interest rate is $1/\beta$. But in a

[22]In particular, it should not be read as saying that $\theta \simeq 0.003$ is a "good" choice (the intersection of the curve and dashed line in Figure 2), simply because the local and global Backus-Smith correlations are similar. This value is *not* independent of other parameters of the calibration, and other equilibrium moments are inconsistent at this value. Even if the Backus-Smith correlation was the only moment of interest, there is no way to know in advance what value of θ replicates the global solution's correlation without already having an accurate, i.e. global, solution.

two-country model it is not, not even on average[23].

3.2.2 A Global Solution

The true model is stationary because the average interest rate in the model is less than $1/\beta$. This is related to home bias in bond holdings: households save in domestic bonds and lend in foreign bonds, as already shown in Figure 1. Because domestic production is a majority of domestic consumption, the domestic price level is more correlated with domestic than foreign shocks. As a result, the real return on foreign bonds (which is susceptible to exchange rate risk) is riskier than that on home bonds. And so borrowers bear more risk than savers. The average interest rate is therefore less than the steady state level to compensate lenders for this extra risk. By ignoring risk, the steady state would eliminate this effect, producing average interest rates that are a too high and nonstationary debt dynamics.

Our solution method is and extension of Maliar and Maliar (2015) and is described in detail in Appendix B. The basic principles are quite straightforward, though. We guess a policy function for each variable in terms of the states[24], simulate the model, and update the policy functions based on the equation errors. Because our solution is global, we do not have to find a steady state first and then approximate the solution nearby. Instead, the algorithm fits an approximation to the entire solution manifold simultaneously. And so the average quarterly interest rate in the solution is less than $1/\beta - 1 = 1\%$. Asset dynamics are therefore stable. A similar effect occurs in Huggett (1993) and Aiyagari (1994) where idiosyncratic risk leads the equilibrium interest rate to be less than the discount rate.

Figure 3 displays the equilibrium relationship between interest rates and asset holdings in

[23]There is one alternative method, suggested by Devereux and Sutherland (2010): guess a mean interest rate, linearize nearby, simulate, update the interest rate guess, and iterate to convergence. However, this will not work in our model. As noted by Rabitsch, Stepanchuk, and Tsyrennikov (2015), the analytic solution for average asset holdings computed by local linearization formulae of Devereux and Sutherland (2011) do not hold away from the riskless interest rate.

[24]The basis functions we use for this are the full set of quadratic functions of the states, although this choice is a matter of accuracy rather than anything more fundamental.

the calibrated solution, when all other variables are at their long-run averages[25]. There are two main points to note. First, that the interest rate is always less than $1/\beta$, stabilizing the model. This holds elsewhere in the state space too; the quarterly average log interest rate is 0.099, very similar to the lines shown in Figure 3. And while the difference between the true average interest rate and the steady state is small - only around one basis point per quarter - the steady state is the knife-edge separating stable from explosive dynamics, so accuracy is essential.

Second, as home's domestic saving increases above its average value (moving to the right in the left-hand panel), the return on home-currency bonds, R^H, falls relative to the return on foreign-currency bonds, R^F. This occurs because more domestic saving by country H requires more foreign borrowing by country F. Because international borrowers are more exposed to risk than savers, households in country F must be compensated through a lower cost of borrowing relative to the return on their assets. So in equilibrium, R_H falls relative to R_F.

3.2.3 Verifying solution accuracy

To check that the global method we develop is indeed more accurate than competing alternatives, we compute the errors on the four Euler equations and compare across solution methods.

Figure 4 shows the distribution of the log errors on the four Euler equations[26]. The blue and red lines show the errors from the linear and quadratic global solutions respectively. The black line shows the errors using the Devereux and Sutherland (2011) method, using an EDF of $\theta = 0.01$ to stationarize the model. Figure 5 summarizes these densities, displaying the mean and mean absolute error, reported in $\log_{10}$ units.

[25]Because equilibrium assets holdings and endowments are highly correlated, the conditional distributions of B_H^H and B_F^F are much narrower than the unconditional distributions shown in Figure 1. The x-axis ranges of the panels in Figure 3 cover more than twice the conditional range of B_H^H and B_F^F observed in a ten-million period simulation. Observations outside these ranges essentially never happen.

[26]Each is labeled with a variable name, because the (Maliar and Maliar, 2015) method associates each equation with a rules for one variable. Here, the Euler equations are satisfied by adjusting the rules for $B_{H,t}^H, \mathcal{E}_t^{H,F}, R_{H,t}$ and $R_{F,t}$

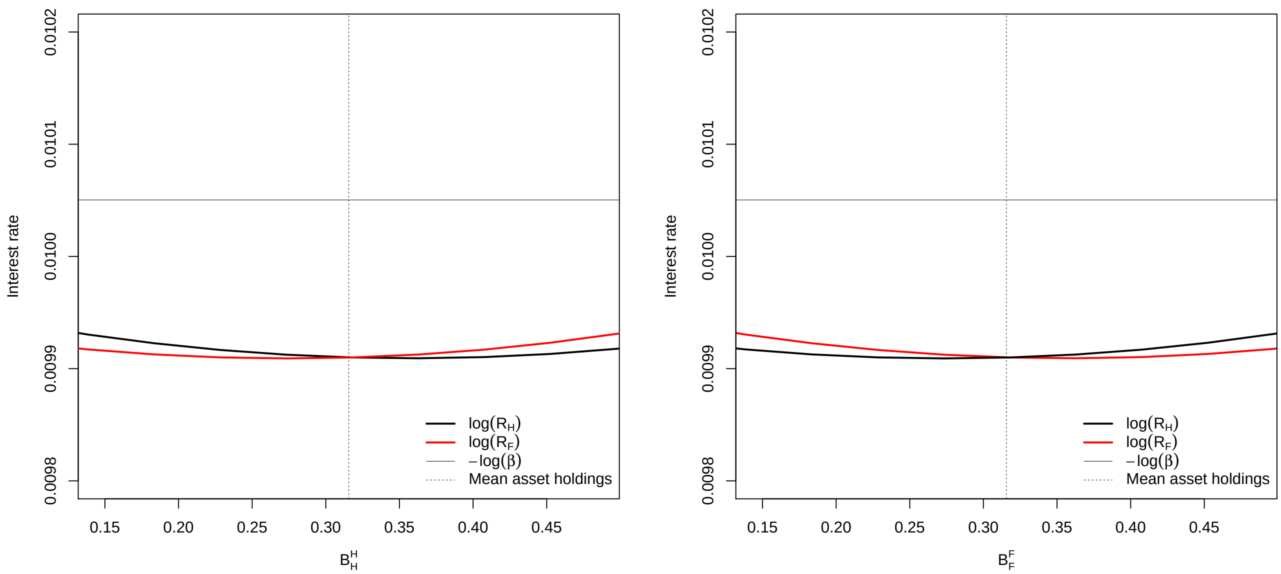

Figure 3: Equilibrium Asset Returns

As these figures makes clear, the local method is biased, as it approximates around a point where the interest rate is $1/\beta$, which is not the true long-run average. This leads to *systematic* errors on the Euler equations, equivalent to nearly two basis points per quarter. This means that in the long run, households are incurring substantial losses relative to their optimal portfolio by following the policy rules computed in the local approximations. Not only is the local solution highly sensitive to changes in the stabilizing assumptions, it is also systematically wrong.

In the global quadratic approach, however, there is almost no bias at all; the average error on the Euler equations is in the order of one tenth to one thousandth of a basis point. The global solution does not suffer from the systematic errors displayed by the local one.

The quadratic global solution method also generates very small absolute equation errors. The results from our method produce absolute log errors on the Euler equations that average to less than 10^{-4}, or below one basis point per quarter. At least on some equations, this is an order of magnitude smaller than the alternative solution methods.

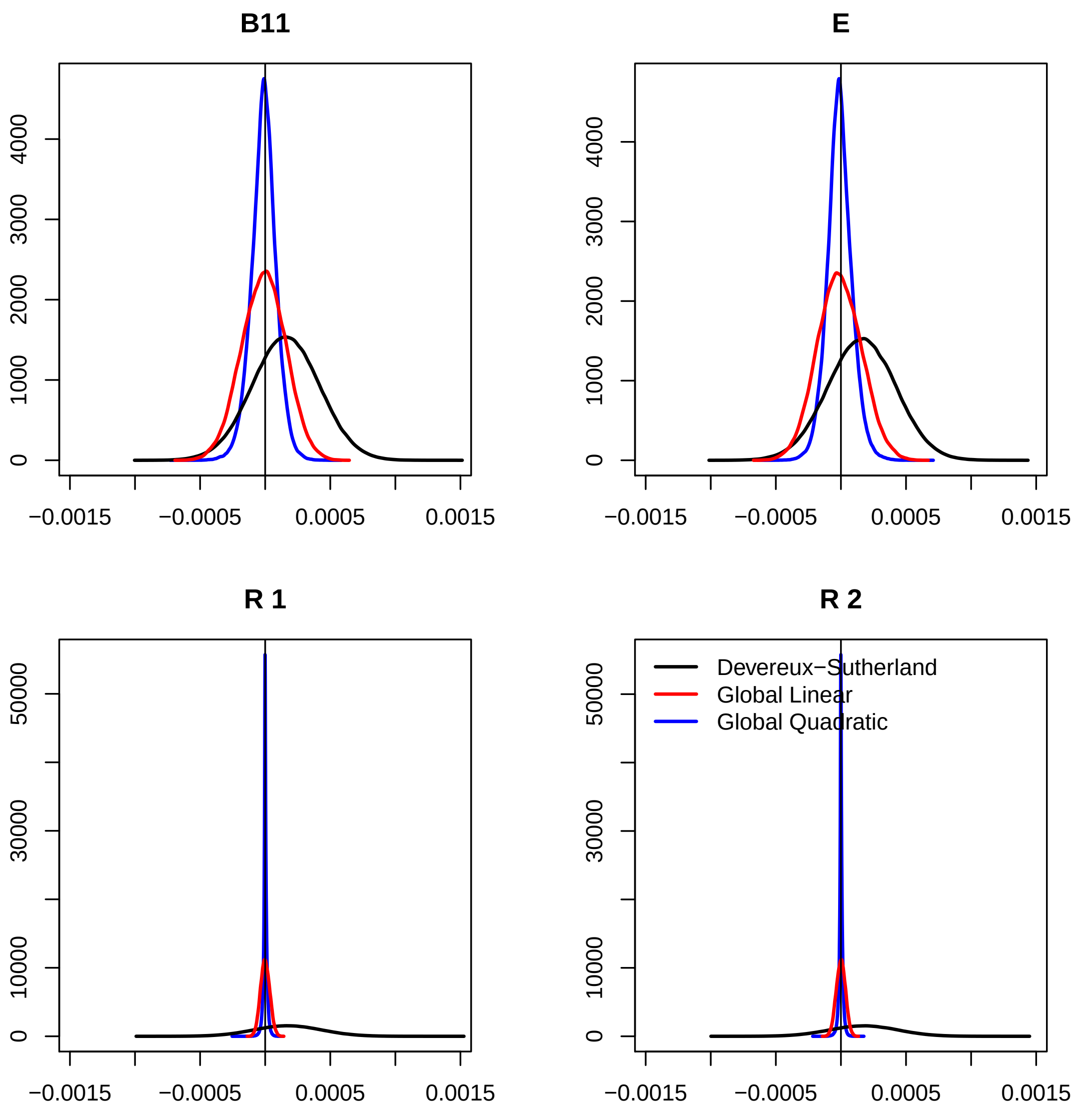

Figure 4: Distribution of log errors on Euler equations

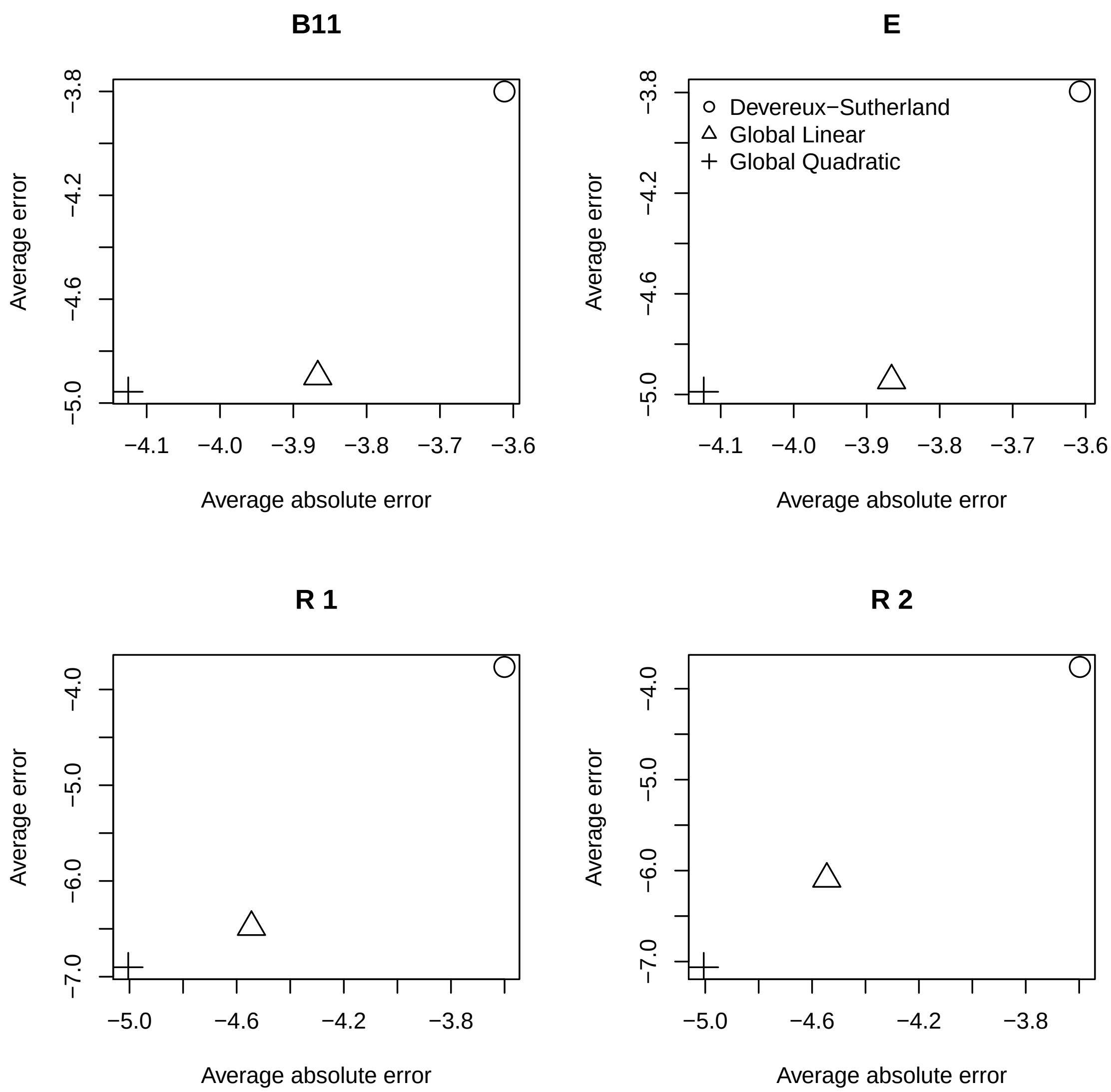

Figure 5: Log errors on Euler equations (expressed in $\log_{10}$ units)

4 Parameter Sensitivity

NOTE: FIGURES OUT OF DATE. UPDATED (BUT QUALITATIVELY UNCHANGED) VERSION COMING SOON.

In this section we examine how the equilibrium of the model varies with the chosen calibration. We find that the equilibrium correlations, especially the Backus-Smith correlation, are highly sensitive to parameter values. This includes parameter values that are not well identified in the literature, such as the risk aversion coefficient γ, and elasticity of substitution between foreign and domestic tradables η.

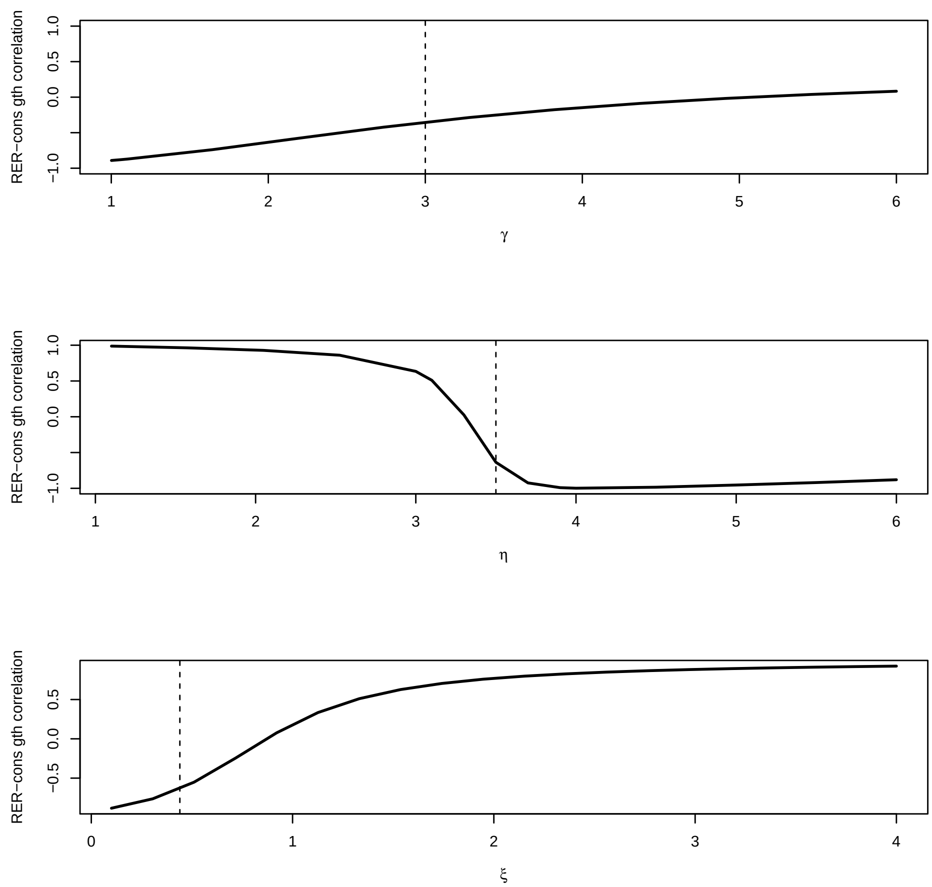

Figure 6: Parameter Sensitivity: Backus-Smith Correlation

The Backus-Smith correlation is increasing in risk aversion, γ (Figure 6). The correlation is nearly negative one when γ approaches log utility, but as risk aversion increases, it pushes the correlation towards zero. Intuitively, as agents become more risk averse, they use their asset portfolios to eliminate risk more aggressively. Unbalanced bond holdings are exposed to real exchange rate risk, because real and nominal rates are correlated (Figure 7), so more risk averse agents choose portfolios that reduce the correlation between real exchange rates and consumption. Agents accomplish this by choosing a more home-biased portfolio (Figure 9), reducing their exposure to exchange rate risk and increasing the exposure to domestic inflation risk. Home biased portfolios lose when there is domestic inflation, so the consumption-inflation correlation falls (Figure 8).

The Backus-Smith correlation is especially sensitive to η, the elasticity of substitution between foreign and domestic tradables. Values of η less than 2, as is commonly assumed in the international macro literature[27], imply that consumption and real exchange rates are almost perfectly correlated, contrary to the data. However, when the elasticity of substitution is larger than 4, as commonly estimated in the trade literature, the correlation is nearly -1.

Why is it so sensitive? A positive productivity shock in country 1 has three effects on relative prices. First, both countries increase consumption of the country H's tradable good, but due to home bias, the price of country H's tradable bundle decreases relative to country F's tradable bundle. The second effect is that the price of the nontradable good relative to the tradable bundle increases in both countries, and increases by more in country H, where the tradable bundle increase is larger due to home bias. The third effect is that the nominal exchange rate rises; the law of one price holds, and the domestic prices of the tradables are fixed, so the exchange rate appreciation depends on the change in relative price of the tradable goods. The real exchange rate $Q_t^{H,F} = \frac{P_{F,t}}{P_{H,t}}\mathcal{E}_t^{H,F}$ is determined by these three effects in the following ways: the first effect increases the real exchange rate by decreasing the relative price

[27] Macroeconomic papers estimate this parameter in a broad range: Taylor (1993) estimates .4, Heathcote and Perri (2002) estimate .9, Whalley (1985) estimates 1.5, and Corsetti, Dedola, and Leduc (2008) estimate 3.3.

of consumption in country H, the second effect decreases the real exchange rate by increasing the relative price of consumption in country F, and the third effect increases the real exchange rate by increasing the nominal exchange rate.

The first and third effects are weakened by a larger η. As domestic and foreign tradables become more substitutable, an increase in one country's tradables has a smaller effect on the relative price of tradables and on the price of the tradable bundle. This allows the second effect to dominate, so that a productivity increase can decrease the real exchange rate. Positive productivity shocks increase income and also consumption for $\gamma > 1$, so a higher elasticity of substitution η reduces the Backus-Smith correlation.

The elasticity of substitution between tradables and nontradables, ξ, is also poorly identified in the literature and has opposite effects as η. When tradables are complements, as commonly assumed, real exchange rates and consumption growth are strongly negatively correlated. But as ξ rises, the correlation turns positive. A higher ξ reduces the second effect mentioned above; the impact of a larger tradable bundle on the price of nontradables is reduced when they are more substitutable. But when they are complements, this effect dominates, and the real exchange rate falls in response to a productivity shock, which also increases consumption. Stockman and Tesar (1995) estimate this parameter to be .44, which is a common calibration. They use both developing and developed countries in their study, which leads to lower estimates than restricting the sample to industrialized countries, as in Mendoza (1991) who estimates an elasticity of .74.

5 Concluding Remarks

We have shown that standard international macroeconomic theory predicts two patterns observed in the data: negative correlation between consumption and real exchange rates and home bias in bonds. These patterns are driven by household portfolio choice, so we could not

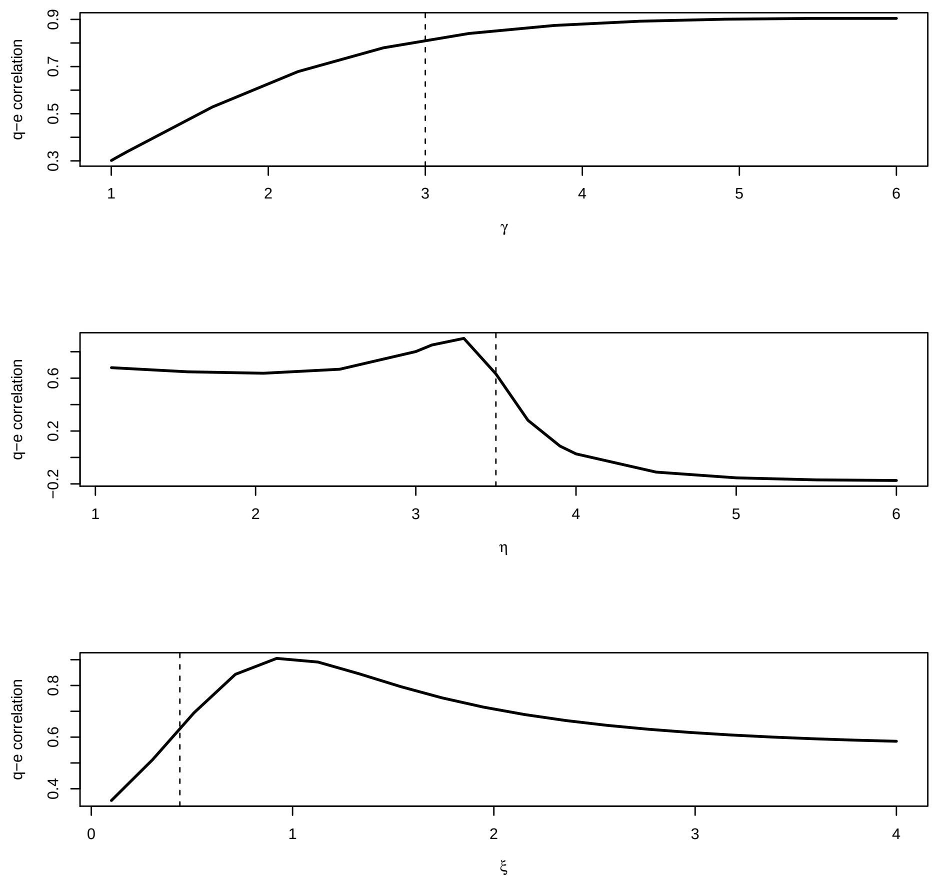

Figure 7: Parameter Sensitivity: Real-Nominal Exchange Rate Correlation

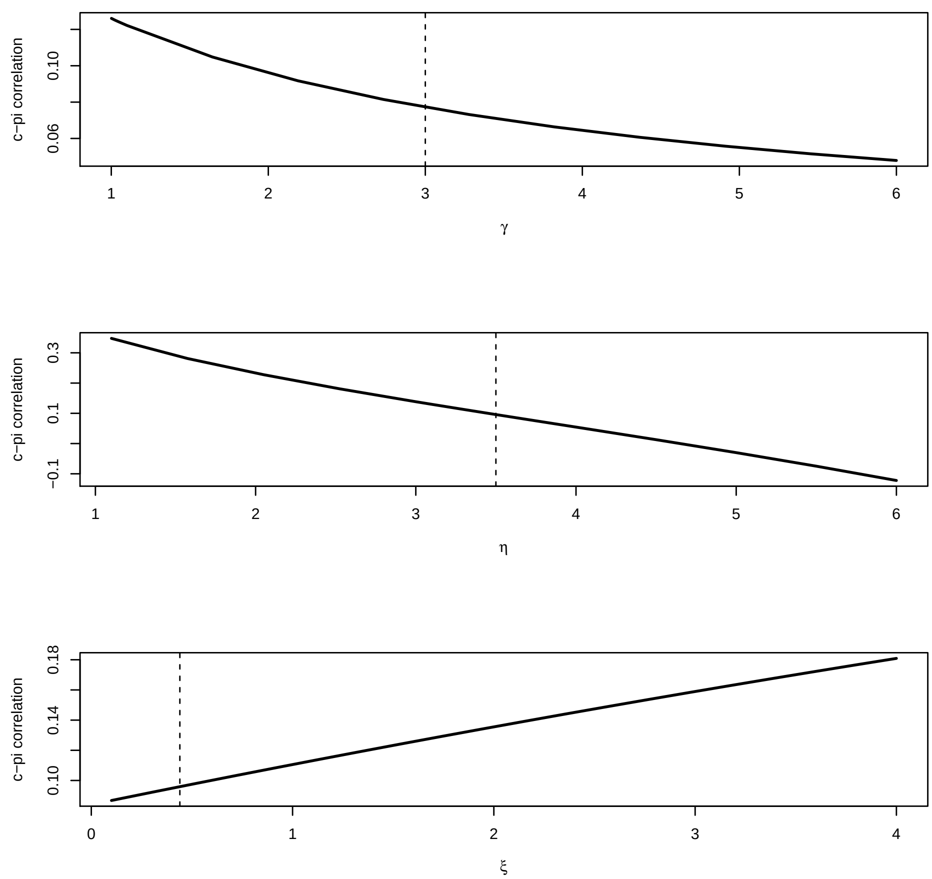

Figure 8: Parameter Sensitivity: Relative Consumption-Inflation Correlation

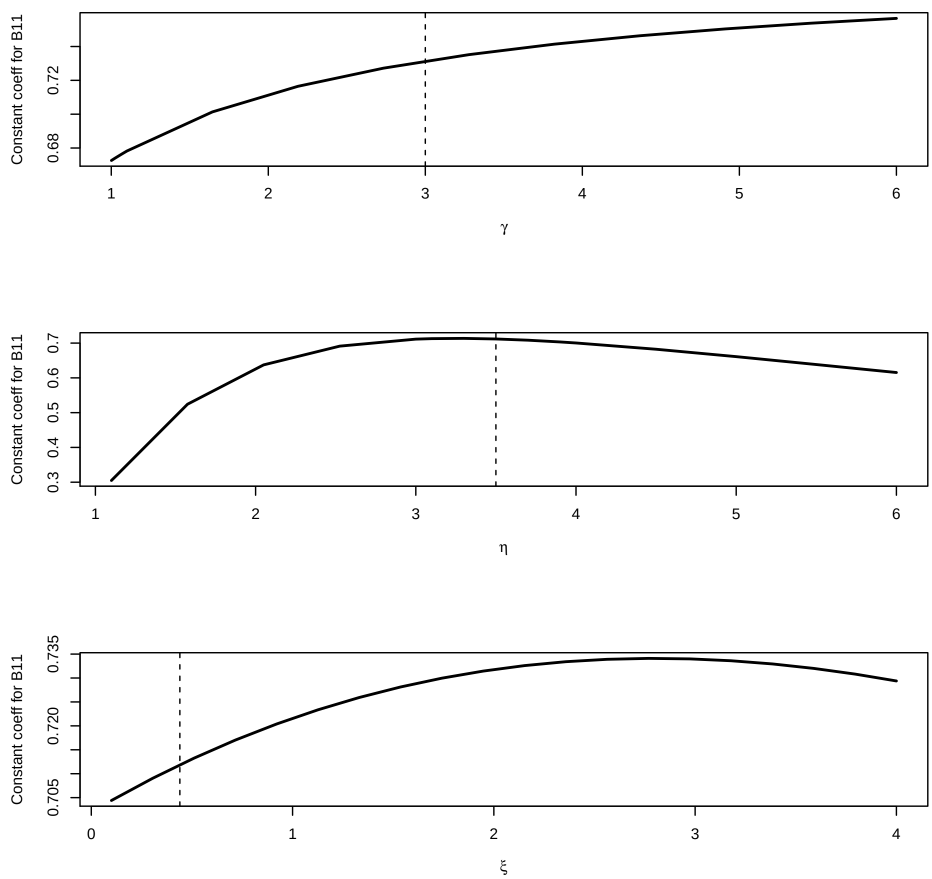

Figure 9: Parameter Sensitivity: Bond Home Bias

use linear methods to solve the model. Instead, we employed a global solutions method which generalizes the projections approach of Maliar and Maliar (2015). We show that our method is highly accurate. We also show that perturbation methods with an ad hoc endogenous discount factor are a poor approximation and are not robust to choice of the EDF parameter. We also show that quantitative results are sensitive to asset market structure, so that models with a meaningful portfolio choice must be solved directly, instead approximated with asset market simplifications.

The portfolio channel driving our results has implications for a wide range of international macro analyses. Any model in which households trade nominal noncontingent bonds is characterized by the optimality condition (28). So if models feature high risk aversion and consumption that covaries more with domestic relative to foreign inflation, they should expect consumption and real exchange rates to be negatively correlated.

Our solutions method may also be useful for further research. There are many cases in which linear methods are insufficient to capture crucial economic forces - particularly when risk and portfolio choice are involved. Our method is general, efficient, and easily implemented, because it generalizes the method of Maliar and Maliar (2015) to the case where control variables cannot be easily substituted out of the equations defining the model. We provide the code and documentation on our websites.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109(3), 659–684.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992): "International Real Business Cycles," *Journal of Political Economy*, 100(4), 745–775.

Backus, D. K., and G. W. Smith (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35(34), 297–316.

Baxter, M., and U. J. Jermann (1997): "The International Diversification Puzzle Is Worse Than You Think," *The American Economic Review*, 87(1), 170–180.

Benigno, G., and H. Kucuk-Tuger (2008): "Financial globalization, home equity bias and international risk-sharing," .

Benigno, G., and C. Thoenissen (2008): "Consumption and real exchange rates with incomplete markets and non-traded goods," *Journal of International Money and Finance*, 27(6), 926–948.

Benigno, P. (2009): "Are valuation effects desirable from a global perspective?," *Journal of Development Economics*, 89(2), 170–180.

Berriel, T. C., and S. Bhattarai (2013): "Hedging Against the Government: A Solution to the Home Asset Bias Puzzle," *American Economic Journal: Macroeconomics*, 5(1), 102–134.

Cavallo, M., and C. Tille (2006): "Current account adjustment with high financial integration: a scenario analysis," *Economic Review*, pp. 31–45.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002): "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *The Review of Economic Studies*, 69(3), 533–563.

Coeurdacier, N., and P.-O. Gourinchas (2016): "When bonds matter: Home bias in goods and assets," *Journal of Monetary Economics*, 82(Supplement C), 119–137.

Coeurdacier, N., R. Kollmann, and P. Martin (2010): "International portfolios, capital accumulation and foreign assets dynamics," *Journal of International Economics*, 80(1), 100–112.

Coeurdacier, N., and H. Rey (2013): "Home Bias in Open Economy Financial Macroeconomics," *Journal of Economic Literature*, 51(1), 63–115.

Colacito, R., and M. M. Croce (2013): "International Asset Pricing with Recursive Preferences," *The Journal of Finance*, 68(6), 2651–2686.

Corsetti, G., L. Dedola, and S. Leduc (2008): "International Risk Sharing and the Transmission of Productivity Shocks," *The Review of Economic Studies*, 75(2), 443–473.

——— (2014): "The International Dimension of Productivity and Demand Shocks in the Us Economy," *Journal of the European Economic Association*, 12(1), 153–176.

Devereux, M. B., and A. Sutherland (2007): "Monetary Policy and Portfolio Choice in an Open Economy Macro Model," *Journal of the European Economic Association*, 5(2-3), 491–499.

——— (2010): "Country portfolio dynamics," *Journal of Economic Dynamics and Control*, 34(7), 1325–1342.

——— (2011): "Country Portfolios in Open Economy Macro-Models," *Journal of the European Economic Association*, 9(2), 337–369.

Engel, C. (1999): "Accounting for U.S. Real Exchange Rate Changes," *Journal of Political Economy*, 107(3), 507–538.

Engel, C. M., and A. Matsumoto (2009): "International Risk Sharing: Through Equity Diversification or Exchange Rate Hedging?," SSRN Scholarly Paper ID 1438847, Social Science Research Network, Rochester, NY.

Evans, M. D. D., and V. Hnatkovska (2012): "A method for solving general equilibrium models with incomplete markets and many financial assets," *Journal of Economic Dynamics and Control*, 36(12), 1909–1930.

Fernndez-Villaverde, J., J. F. Rubio-Ramrez, and F. Schorfheide (2016): "Chapter 9 - Solution and Estimation Methods for DSGE Models," in *Handbook of Macroeconomics*, ed. by J. B. T. a. H. Uhlig, vol. 2, pp. 527–724. Elsevier, DOI: 10.1016/bs.hesmac.2016.03.006.

Gourinchas, P.-O., and H. Rey (2014): "External Adjustment, Global Imbalances, Valuation Effects," *Handbook of International Economics*, 4, 585–645.

Heathcote, J., and F. Perri (2002): "Financial autarky and international business cycles," *Journal of Monetary Economics*, 49(3), 601–627.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control*, 17(5), 953–969.

Judd, K. L. (1998): *Numerical methods in economics*. MIT press.

Karabarbounis, L. (2014): "Home production, labor wedges, and international business cycles," *Journal of Monetary Economics*, 64, 68–84.

Lane, P. R., and J. C. Shambaugh (2010a): "Financial Exchange Rates and International Currency Exposures," *The American Economic Review*, 100(1), 518–540.

——— (2010b): "The long or short of it: Determinants of foreign currency exposure in external balance sheets," *Journal of International Economics*, 80(1), 33–44.

Lee, J., F. Ghironi, and A. Rebucci (2009): "The Valuation Channel of External Adjustment," IMF Working Papers 09/275, International Monetary Fund.

Lewis, K. K. (1999): "Trying to Explain Home Bias in Equities and Consumption," *Journal of Economic Literature*, 37(2), 571–608.

Lucas Jr., R. E. (1982): "Interest rates and currency prices in a two-country world," *Journal of Monetary Economics*, 10(3), 335–359.

Lustig, H., and A. Verdelhan (2016): "Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates?," *Working Paper*.

Maggiori, M., B. Neiman, and J. Schreger (2017): "International Currencies and Capital Allocation," *Working Paper*.

Maliar, L., and S. Maliar (2015): "Merging simulation and projection approaches to solve high-dimensional problems with an application to a new Keynesian model," *Quantitative Economics*, 6(1), 1–47.

Matsumoto, A., and C. Engel (2009): *The international diversification puzzle when goods prices are sticky: it's really about exchange-rate hedging, not equity portfolios*, no. 9-12. International Monetary Fund.

Mendoza, E., V. Quadrini, and J. RiosRull (2009): “Financial Integration, Financial Development, and Global Imbalances,” *Journal of Political Economy*, 117(3), 371–416.

Mendoza, E. G. (1991): “Real Business Cycles in a Small Open Economy,” *The American Economic Review*, 81(4), 797–818.

Mukhin, D., O. Itskhoki, and others (2016): “Exchange Rate Disconnect,” in *2016 Meeting Papers*. Society for Economic Dynamics.

Rabitsch, K., S. Stepanchuk, and V. Tsyrennikov (2015): “International portfolios: A comparison of solution methods,” *Journal of International Economics*, 97(2), 404–422.

Rahbari, E. (2009): “Portfolios and business cycles in an open economy dsge model,” *manuscript, London Business School*.

Schmitt-Grohe, S., and M. Uribe (2003): “Closing small open economy models,” *Journal of International Economics*, 61(1), 163–185.

Stockman, A. C., and L. L. Tesar (1995): “Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements,” *The American Economic Review*, 85(1), 168–185.

Taylor, J. B. (1993): *Macroeconomic policy in a world economy: from econometric design to practical operation*. WW Norton.

Tille, C. (2008): “Financial integration and the wealth effect of exchange rate fluctuations,” *Journal of International Economics*, 75(2), 283–294.

Tille, C., and E. van Wincoop (2010): “International capital flows,” *Journal of International Economics*, 80(2), 157–175.

Whalley, J. (1985): *Trade liberalization among major world trading areas*. MIT Press.

A Approximations

Any asset with return X_{t+1} satisfies the pricing equation for the household in country i:

$$1 = \beta \mathbb{E}_t \left(X_{t+1} \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} \right) \tag{30}$$

Define $c_{i,t+1} = \log \left(C_{i,t+1}/C_{i,t} \right)$, $x_{t+1} = \log X_{i,t+1}$, $\bar{x}_t = \mathbb{E}_t x_{t+1}$, and $\bar{c}_t = \mathbb{E}_t c_{t+1}$. Then the second-order expansion of the integrand in the pricing equation is:

$$\begin{aligned} X_{t+1} \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} &= e^{x_t - \gamma c_{i,t+1}} \\ &= e^{\bar{x}_{t+1} - \gamma \bar{c}_t} \left[\begin{array}{r} 1 + (x_{t+1} - \bar{x}_t) - \gamma (c_{i,t+1} - c_{i,t}) + \frac{1}{2}(x_{t+1} - \bar{x}_t)^2 \\ + \frac{\gamma^2}{2}(c_{i,t+1} - \bar{c}_t)^2 - \gamma (x_{t+1} - \bar{x}_t)(c_{i,t+1} - \bar{c}_t) \end{array} \right] \end{aligned}$$

Substituting into (30) gives

$$\begin{aligned} \beta^{-1} e^{-(\bar{x}_t - \gamma \bar{c}_t)} &= 1 + \frac{1}{2} \mathrm{Var}_t \; x_{t+1} + \frac{\gamma^2}{2} \mathrm{Var}_t \; c_{i,t+1} + \gamma \mathrm{Cov}_t (x_{t+1}, c_{i,t+1}) \\ &\simeq \exp \left(\frac{1}{2} \mathrm{Var}_t \; x_{t+1} + \frac{\gamma^2}{2} \mathrm{Var}_t \; c_{i,t+1} + \gamma \mathrm{Cov}_t (x_{t+1}, c_{i,t+1}) \right) \\ \Rightarrow \qquad - \log \beta &\simeq \bar{x}_t - \gamma c_{i,t} + \frac{1}{2} \mathrm{Var}_t \; x_{t+1} + \frac{\gamma^2}{2} \mathrm{Var}_t \; c_{i,t+1} + \gamma \mathrm{Cov}_t (x_{t+1}, c_{i,t+1}) \end{aligned}$$

For the foreign bond,

$$X_t = \frac{1}{R^j_{t+1}} \frac{\mathcal{E}^{ij}_{t+1}}{\mathcal{E}^{ij}_t} \frac{P^i_t}{P^i_{t+1}}$$

Let $r^j_{t+1} = \log R^j_{t+1}$, $e^{ij}_{t+1} = \log \left(\mathcal{E}^{ij}_{t+1}/\mathcal{E}^{ij}_t \right)$, and $\pi_{i,t+1} = \log \left(P^i_{t+1}/P^i_t \right)$. Then:

$$\mathrm{Var}_t x_{t+1} = \mathrm{Var}_t e^{ij}_{t+1} + \mathrm{Var}_t \pi_{i,t+1} - \mathrm{Cov}_t (e^{ij}_{t+1}, \pi_{i,t+1})$$

Substituting in for this then gives equation (24)

$$\begin{aligned} 0 \approx \log R^j_{t+1} + \log \beta + E_t e^{i,j}_{t+1} - \gamma E_t c_{i,t+1} - E_t \pi_{i,t+1} + \frac{Var_t(e^{i,j}_{t+1})}{2} + \gamma^2 \frac{Var_t(c_{i,t+1})}{2} + \frac{Var_t(\pi_{t+1})}{2} \\ - \gamma Cov_t(e^{i,j}_{t+1}, c_{i,t+1}) - Cov_t(e^{i,j}_{t+1}, \pi_{i,t+1}) + \gamma Cov_t(c_{t+1}, \pi_{i,t+1}) \end{aligned} \tag{31}$$

Likewise for the domestic Euler equation, we recover equation (23):

$$0 \approx \log R^i_{t+1} + \log\beta - \gamma E_t c_{i,t+1} - E_t\pi_{i,t+1} + \gamma^2 \frac{Var_t(c_{i,t+1})}{2} + \frac{Var_t(\pi_{i,t+1})}{2} + \gamma Cov_t(c_{i,t+1}, \pi_{i,t+1}) \tag{32}$$

B Computation

B.1 Model Specification

The method we develop can be used to solve a general model with exogenous state X_t, endogenous state Y_t and controls $C_t = (C^1_t, C^2_t)$, where the equilibrium conditions can be written as:

$$\begin{aligned}
X_{t+1} &= \Theta(X_t, \epsilon_{t+1}) & \epsilon_{t+1} \sim \mathcal{N}(0, \Sigma)\\
Y_t &= f_1\left(Y_{t-1}, X_t, C_t, Y_t, \mathbb{E}_t g_1\left(X_{t+1}, C_{t+1}\right)\right)\\
C^1_t &= f_2\left(Y_{t-1}, X_t, C_t, Y_t, \mathbb{E}_t g_2\left(X_{t+1}, C_{t+1}\right)\right)\\
C^2_t &= h(Y_{t-1}, X_t, Y_t, C_t)
\end{aligned}$$

Here, the partition of the control space allows for more model descriptions with more forward-looking equations than endogenous states, as is the case ni our model when $B_{21,t}$ and $B_{12,t}$ are written out via the market clearing conditions. A solution to this model is a pair of functions $Y_t = y(X_t, Y_{t-1})$ and $C_t = c(X_t, Y_{t-1})$ satisfying:

$$\begin{aligned}
y(X_t, Y_{t-1}) &= f_1\left(Y_{t-1}, X_t, c(X_t, Y_{t-1}), y(X_t, Y_{t-1}), \mathbb{E}_t g_1\left(X_{t+1}, c(X_{t+1}, y(X_t, Y_{t-1}))\right)\right)\\
c^1(X_t, Y_{t-1}) &= f_2\left(Y_{t-1}, X_t, c(X_t, Y_{t-1}), y(X_t, Y_{t-1}), \mathbb{E}_t g_2\left(X_{t+1}, c(X_{t+1}, y(X_t, Y_{t-1}))\right)\right)\\
c^2(X_t, Y_{t-1}) &= h(Y_{t-1}, X_t, y(X_t, Y_{t-1}), c(X_t, Y_{t-1}))
\end{aligned}$$

Where $C(\cdot) = (c^1(\cdot), c^2(\cdot))$. We can write the model in this paper in this form by letting:

$$
\begin{aligned}
X_t &= (a_t^1, a_t^2) \\
Y_t &= (B_{1,t+1}^1, B_{2,t+1}^2) \\
C_t^1 &= (r_{t+1}^1, r_{t+1}^2) \\
C_t^2 &= (c_{1,t}, c_{2,t}, x_{1,t}^1, x_{2,t}^2, x_{1,t}^2, x_{2,t}^1, n_{1,t}, n_{2,t}, p_{1,t}, p_{2,t}, p_{1,t}^2, p_{2,t}^1, e_t^{12})
\end{aligned}
$$

Where lower case variables are the logarithms of their upper case counterparts (this is simply a change of variables that improves the accuracy of the resulting solution). The forward-looking equations for the endogenous states are defined by:

$$
\begin{aligned}
f_1\left(Y_{t-1}, X_t, C_t, Y_t, \mathbb{E}g_1\right) &= \begin{bmatrix} B_{1,t+1}^1 - \left(\gamma c_{2,t} - \rho + r_{1,t+1} + e_t^{12} + p_{2,t} + \log \mathbb{E}g_1^1\right) \\ B_{2,t+2}^1 - \left(\gamma c_{1,t} - \rho + r_{2,t+1} - e_t^{12} + p_{1,t} + \log \mathbb{E}g_1^2\right) \end{bmatrix} \\
g_1(X_{t+1}, C_{t+1}) &= \begin{bmatrix} \exp\left(-e_t^{12} - (\gamma c_{2,t+1} + p_{2,t+1})\right) \\ \exp\left(e_t^{12} - (\gamma c_{1,t+1} + p_{1,t+1})\right) \end{bmatrix}
\end{aligned}
$$

So these reduce to $B_{1,t+1}^1 = B_{1,t+1}^1$ and $B_{2,t+1}^2 = B_{2,t+1}^2$ if and only if the two international Euler equations hold. For the forward-looking controls:

$$
\begin{aligned}
f_1\left(Y_{t-1}, X_t, C_t, Y_t, \mathbb{E}g_1\right) &= \begin{bmatrix} \rho - \gamma c_{1,t} - p_{1,t} - \log \mathbb{E}g_2^1 \\ \rho - \gamma c_{2,t} - p_{2,t} - \log \mathbb{E}g_2^2 \end{bmatrix} \\
g_1(X_{t+1}, C_{t+1}) &= \begin{bmatrix} \exp\left(-\left(\gamma c_{1,t+1} + p_{1,t+1}\right)\right) \\ \exp\left(-\left(\gamma c_{2,t+1} + p_{2,t+1}\right)\right) \end{bmatrix}
\end{aligned}
$$

And the function $h(\cdot)$ is given by the remaining contemporaneous identities.

B.2 Functional forms

We approximate the functional solutions $y(X, Y)$ and $c(X, Y)$ with polynomials. The notation $d = \phi(\delta)$ means that the function $d(X, Y)$ is defined by the sum over the polynomials of (X, Y)

with weights given by the elements of the vector $\boldsymbol{\delta}$. That is:

$$d(X,Y) = \sum_i \delta_i q_i^n((X,Y))$$

Where $q^n(\mathbf{z})$ is the vector of all possible n^{th}-order combinations of the elements of $\mathbf{z}$. For example, if $\mathbf{z} = (x,y) \in \mathbb{R}^2$, then:

$$q^2(\mathbf{z}) = (1, x, y, x^2, xy, y^2)$$

So the approximate solutions for $y(X,Y)$ and $c(X,Y)$ can be described by vectors $\boldsymbol{\delta}^y$ and $\boldsymbol{\delta}^c$[28].

In our calculations we use standard polynomials to approximate the functional forms of the solution, but in general one can use Chebychev or other polynomials as instead (indeed, these often have superior numerical properties). We also scale the basis polynomials depending on the expected variance of the states.

B.3 The algorithm

Given guesses $\boldsymbol{\delta}^y_j$ and $\boldsymbol{\delta}^c_j$, the instructions for updating them to coefficient guesses $\boldsymbol{\delta}^y_{j+1}$ and $\boldsymbol{\delta}^c_{j+1}$ are:

1. Simulate $(X_{t-1}, Y_{t-1}, X_t, Y_t)$ for a very large number of points. In the paper we use a simulation of 1,000,000 points, discarding the first 10,000 periods.

2. Use the EDS algorithm of Maliar and Maliar (2015), reduce this to an almost-ergodic set of many fewer points, denoted $\xi^j = \{X^j_\ell, Y^j_\ell, X^{j'}_\ell, Y^{j'}_\ell\}_{\ell=1}^m$. In the paper we use a target of $m = 1000$ points.

3. Solve for the set $\{\tilde{C}^2_{j,\ell}\}_{\ell=1}^m$ satisfying $\tilde{C}^2_{j,\ell} = h(Y^j_\ell, X^{j'}_\ell, Y^{j'}_\ell, \tilde{C}_{j,\ell})$

4. Regress the $\{\tilde{C}^2_{j,\ell}\}_{\ell=1}^m$ on $q^2(Y^j_\ell, X^{j'}_\ell)$ to get updated coefficients $\boldsymbol{\delta}^{c,2}_{j+1}$.

[28] The same notation can be easily extended to vector-valued functions $y(X,Y)$ and $c(X,Y)$ by stacking the coefficients into the vectors $\boldsymbol{\delta}^y$ and $\boldsymbol{\delta}^c$.

5. Using monomial integration to evaluate the integrals, solve for $\{\tilde{Y}_\ell^{j'}, \tilde{C}_{j,\ell}^1\}_{\ell=1}^m$ solving:

$$\tilde{Y}_\ell^{j'} = f_1\left(Y_\ell^j, X_\ell^{j'}, \hat{c}_{j+1}(X_\ell^{j'}, Y_\ell^j), \tilde{Y}_\ell^{j'}, \mathbb{E}_t g_1\left(X_{t+1}, \hat{c}_{j+1}(X_t^{j'}, \tilde{Y}_\ell^{j'})\right)\right)$$
$$\tilde{C}_{j,\ell}^1 = f_1\left(Y_\ell^j, X_\ell^{j'}, \hat{c}_{j+1}(X_\ell^{j'}, Y_\ell^j), \tilde{Y}_\ell^{j'}, \mathbb{E}_t g_1\left(X_{t+1}, \hat{c}_{j+1}(X_t^{j'}, \tilde{Y}_\ell^{j'})\right)\right)$$

6. Regress $\{\tilde{Y}_\ell^{j'}, \tilde{C}_{j,\ell}^1\}_{\ell=1}^m$ on $q^2(Y_\ell^j, X_\ell^{j'})$ to get updated coefficients $\boldsymbol{\delta}_{j+1}^y, \boldsymbol{\delta}_{j+1}^{c,1}$.

The preceding description outlines the simplest way to update the vectors of coefficients that describe the model solution. However, in practice, a number of additional complications arise, omitted from the algorithm description for clarity. The first of these is the stopping rule. In the implementation used in the paper we terminate the algorithm when successive updates of the solution coefficients are sufficiently close. However, the ultimate test of convergence is the error on the model equations. With a novel method, such as this, evaluating the solution errors is of primary importance. Hence further discussion of them is promoted to the main body of the text.

Also omitted from this description is the role of damping. The method is, at heart, a policy function iteration algorithm. As is well know, policy function iteration often requires very heavy damping in order to converge, as discussed in Judd (1998). When solving the model in this paper, the initial gain is set to between 0.02 and 0.5 depending on the particular problem. Finally, to partially offset the slow convergence induces by damped policy function iteration we also allow for an adaptive gain. By making the gain a decreasing function of the distance between successive coefficient updates, we reduce the extent of the damping as the model converges.

Because solution can be expressed as a coefficient vector, this method also enables the enforcement symmetry of symmetry. We simply identify the symmetry restrictions that we wish to enforce and average out any deviations in the coefficients from symmetry. This also diminishes error propagation - small solution asymmetries can quickly lead to divergence.

To aid performance, we also write most of the solution code in C++. This leads to fairly swift solutions. A modern laptop takes approximately 2 minutes to produce a second-order solution to the four-state model in this paper. Linear solutions are much faster, but (as discussed in the body of the paper) are also less accurate, producing larger average Euler equation errors.